The Journal of Risk and Insurance

 Vol. 83, No. 3, 519–555 (2016).
DOI: 10.1111/jori.12061

The Sluggish and Asymmetric Reaction of Life Annuity Prices to Changes in Interest Rates

Narat Charupat
Mark J. Kamstra
Moshe A. Milevsky

Abstract

Many assume that in the short run, annuity prices promptly and efficiently respond to changes in interest rates. Using a unique database of quotes, we show this is not the case. Prices are less sensitive to changes in rates than expected, and responses are asymmetric. Prices react more rapidly and with greater sensitivity to an increase than to a decrease in rates. The results are robust, but there is a small degree of heterogeneity in the responses of different insurance companies. When rates increase, larger firms are slightly quicker to improve prices. The opposite is true when rates decline. In sum, we show that the microstructure of annuity dynamics is more complicated than (simply) adding mortality credits to bond yields.

Introduction

It is well known in the literature that prices of life annuities depend on two major factors—the concurrent term structure of interest rates and mortality probabilities (e.g., Dickson, Hardy, and Waters, 2009). A change in the value of either factor will cause prices to adjust. If the annuity markets were frictionless, arbitrage would ensure that the price adjustment will be prompt and proportionate to the change in the factors. In reality, however, there are market frictions such as the person-specific nature of life annuity contracts and the inability to short sell them, and so the adjustment can be slow and/or incomplete.

Narat Charupat is at the DeGroote School of Business, McMaster University. Charupat can be contacted via e-mail: charupat@univmail.cis.mcmaster.ca. Moshe A. Milevsky is at the Schulich School of Business, York University. Milevsky can be contacted via e-mail: milevsky@yorku.ca. Mark J. Kamstra is at the Schulich School of Business, York University. Kamstra can be contacted via e-mail: MKamstra@schulich.yorku.ca. We are grateful for the helpful suggestions of the editor, Keith Crocker, and an anonymous referee, as well as Timothy (Jun) Lu, Sylvaini Benoit, participants at the IFID Centre conference, Financial Management Association Asian Conference, Northern Finance Association 2013, Longevity 8 Conference, City University London and seminar participants at the University of Vienna, York University, as well as Simon Dabrowski and Yue Zhang for research assistance. We thank the Social Sciences and Humanities Research Council of Canada for financial support. Any remaining errors are our own. A previous version of this article was titled "The Annuity Duration Puzzle."

In this article, we use a unique database to examine the sensitivity of life annuity prices to changes in interest rates. The database consists of over 3 million annuity quotes from 25 U.S. annuity providers over the period from September 2004 to May 2012. These are weekly quotes for single-premium immediate life annuities (i.e., ones whose purchase prices are paid once at the start, and whose income payments start right away), and are classified along several dimensions such as ages of annuitants, gender, and guarantee periods. To our knowledge, the database is the most comprehensive and accurate collection of annuity quotes available. Since annuity providers can change their quotes continuously or as conditions warrant, the weekly observations allow us to track closely changes that occurred in the sample period.

We create a measure for the sensitivity of annuity prices to changes in interest rates by deriving an expression for the duration of life annuities. The logic for this expression is akin to that for (modified) durations of bonds in the fixed-income literature, which measures how sensitive bond prices are to changes in yields.[1] The expression is then used to calculate theoretical durations of life annuities, which predicts how responsive annuity prices should be to movements in interest rates in a frictionless market.[2] Next, we empirically estimate the durations of life annuities, using regression analysis. Finally, we compare the estimated empirical durations to the theoretical durations to determine whether real-life annuity prices behave as predicted.

Our findings show that annuity prices do not promptly and fully respond to changes in interest rates. Annuity prices are very insensitive to interest rate movements, especially when price responses are measured over a short time period such as 1 or 2 weeks. More interestingly, we find that the responses are asymmetric. Annuity prices react with greater sensitivity to an increase in interest rates than to a decrease in interest rates. That is, when rates increase, insurance companies reduce their annuity prices by a larger magnitude than what they do in the opposite direction when rates declined.

One possible explanation for the observed sluggish responses is that insurance companies may want to smooth out the price changes and/or wait in order to get a better sense of the trend of interest rates. It is also possible that insurance companies from time to time have unbalanced books (i.e., mismatches of durations of assets and liabilities). These companies may deliberately offer uncompetitive annuity prices or delay price adjustments to allow themselves time to rebalance their books, resulting in the sluggish responses that we observe.[3]

[1]The concept of duration for life annuities is not new, and insurance companies have been using duration measures in their risk management process for decades (see Reitano, 1992, for additional information and references). However, to our knowledge, this article is the first to estimate empirical durations of life annuities and compare them to their theoretical values.

[2]Since changes in mortality probabilities occur very gradually, in the short run, it can be assumed that changes in annuity prices are explained exclusively by movements in interest rates.

[3]This explanation is based on our conversation with insurance companies. We note, however, that it is difficult to verify this information because it would require detailed knowledge of insurance companies' holdings, which are not public information.

The observed asymmetric responses can be due to the providers' reluctance to raise their prices for fear of negative reactions from annuity buyers. The negative customer reaction hypothesis of Rotemberg (1982) suggests that customers prefer relatively stable prices and will react more negatively to unfavorable price changes than to favorable ones.[4] As a result, insurance companies will try to delay a price increase as long as they can. In the meantime, they absorb the loss through the profit margin that they build into the annuity prices.

The results are robust with respect to annuitant purchase ages, lengths of guarantee periods, proxies for interest rates, subsets of annuity providers, and the subperiods of the sample.[5] As a by-product of our robustness tests, we uncover a small degree of heterogeneity in the responses of different individual annuity providers. For example, when interest rates increase, larger firms are slightly quicker to adjust (i.e., reduce) their prices than smaller firms do. The opposite is true when interest rates decline. In addition, firms with higher credit ratings are slightly quicker to adjust to changes in market interest rates, regardless of the direction. Also, lower-rated providers adjust their prices in a more complete manner only when measured over longer periods. Nevertheless, these cross-sectional differences are not large enough to affect our overall findings.

Our findings have implications for previous studies in the literature. For example, the sluggish response suggests that the timing of an annuity purchase can affect the individual's optimal allocation decisions and the magnitude of welfare gains from annuitization. This is particularly important considering that an annuity purchasing decision is irreversible and typically involves a substantial portion of an individual's wealth. Also, caution should be exercised when attempting to infer mortality expectations from observed annuity price as the timing of the calculations matter. Finally, if a pension from a defined benefit plan is to be properly valued as if it were a life annuity, then the lag and asymmetry we identify will affect the valuation. Our focus here, however, is to demonstrate how annuity prices behave in response to interest rate changes. That is, our findings are a first step in an attempt to understand the microstructure dynamics of annuity prices—a subject that, to our knowledge, has not been examined before.[6]

[4]The customer reaction hypothesis is proposed by Rotemberg (1982) in the sticky price literature. Rotemberg also develops a model of sticky prices and tests this model on microeconomic data.

[5]Recently, Koijen and Yogo (2012) report that insurance companies sold life insurance policies and life annuities at "fire sale" prices during the financial crisis of 2008 in order to raise capital. We show later in the article that our results are not qualitatively affected by the unusual pricing during these few months.

[6]The closest mention that we can find is in Cannon and Tonks (2009) who use monthly data to calculate the correlation between UK annuity rates and the UK government 10-year bond yields. They report that the correlation was very high (i.e., 0.98) during the period from 1994 to 2000, but declined substantially to only 0.57 during the period from 2001 to 2007. The low correlation in the latter period (which overlaps with our sample period) is consistent with our findings here.

The article is organized as follows. In the next section, we present a brief review of the relevant literature. Then in "The Pricing and Duration of a Life Annuity" section, we discuss the annuity pricing equation and the expression for the duration of a life annuity, which is our measure of annuity price sensitivity. In "Data Description and Summary Statistics" section we discuss our database. The "Sensitivity Estimations" section presents our empirical results. In the "Robustness Checks" section, we perform a variety of robustness tests. The "Conclusion and Implications" section concludes the article.

Relevant Literature

The study of life annuities consists of four major strands. The first strand examines the optimal portfolio decisions in the presence of life annuities and the welfare gains from annuitization. Yaari (1965) is the first person to extend the life-cycle model (Modigliani and Brumberg, 1954) to include an uncertainty in the length of life. He shows that under some restrictive assumptions, risk-averse individuals should annuitize all of their wealth at retirement provided that annuity prices are actuarially fair. Subsequent studies extend Yaari's model to incorporate more realistic assumptions such as incomplete markets and the irreversibility of annuity purchase. These studies derive the conditions under which full/partial annuitization is optimal and also the optimal timing of annuity purchase. See, for example, Brown, Mitchell, and Poterba (2001), Davidoff, Brown, and Diamond (2005), Milevsky and Young (2007b), Horneff et al. (2009), and Maurer et al. (2013). This strand is obviously related to the pricing of mortality contingent claims, such as discussed in Deng, Brockett, and MacMinn (2012) as well as Dawson et al. (2010).

The second, and closely related, strand of studies attempts to explain the so-called annuity puzzle, which refers to the observation that in real life, voluntary demand for life annuities is much lower than predicted by the models, in terms of both the number of people who purchase life annuities and the value of the transactions. Various reasons have been offered, including strong bequest motives, habit formation in preferences, housing, preannuitized wealth (e.g., private DB pensions and social security benefits), medical expense uncertainty, and behavioral factors. In this strand, see Friedman and Warshawsky (1990), Brown, Mitchell, and Poterba (2001), Dushi and Webb (2004), Turra and Mitchell (2004), Davidoff (2009), Ameriks et al (2011), Benartzi, Previtero, and Thaler (2011) or Inkmann, Lopes, and Michaelides (2011). Likewise, see Scott, Watson, and Hu (2011) for a discussion of how to design annuity products that might be more appealing and thus reduce the magnitude of the observed annuity puzzle.

The third strand of the literature estimates the "money's worth" of life annuities, which is the expected present value of the annuity payments taking into account the term structure of interest rates and mortality probabilities. That is, the money's worth calculations attempt to estimate the annuities' actuarially fair values so that they can be compared with the purchase prices. Typically, the comparison is done in terms of the ratio between the estimated fair value and the purchase price. The calculations have been done for various markets including the U.S., UK, Canadian and German markets (see, e.g., Mitchell et al., 1999; Finkelstein and Poterba, 2004; Cannon and

Tonks, 2009). The findings show that the money's worth ratios are generally only slightly less than unity. This is particularly true when the mortality probabilities used in the calculations reflect the adverse-selection problem that is common in voluntary annuity purchases (i.e., voluntary annuity buyers tend to live longer than the average population). In other words, the evidence suggests that markups (or loadings) on life annuity policies do not appear to be excessive.

Finally, the fourth strand applies annuity pricing techniques to the valuation of corporate and governmental DB pension liabilities, which can be thought of as deferred life annuities (i.e., annuities whose payments do not commence immediately, but rather when the employees retire). Example of these studies include Bodie (1990), Sundaresan and Zapatero (1997), Ippolito (2002), Brown and Wilcox (2009), and Novy-Marx and Rauh (2011). Likewise, any attempt to analyze the optimal timing of Social Security (pension) benefits, such as Sun and Webb (2011) for example, would also require some sort of model for the evolution of pricing.

Many of the above studies explicitly or implicitly assume that life annuities are priced based on the concurrent term structure of interest rates and mortality probabilities. We will shortly show that real-life annuity prices respond sluggishly and asymmetrically to changes in interest rates, which is inconsistent with that assumption.[7]

THE PRICING AND DURATION OF A LIFE ANNUITY

Consider a single-life immediate annuity that pays a lifetime income of \$1 per year (in continuous time), starting immediately and lasting as long as the holder is alive, with a guarantee period of g years. The actuarially fair price of this annuity for an x-year-old individual is:[8]

$$a(x, g, r) = \int_0^g \mathrm{e}^{-rt}\,\mathrm{d}t + \int_g^\infty \mathrm{e}^{-rt} p(x, t)\,\mathrm{d}t, \tag{1}$$

where $p(x, t)$, $0 \leq p(x, t) \leq 1$ is the probability that an individual who is now x years old will survive to at least age $x + t$, and r is the rate of return, which is assumed to be constant for all terms.[9] This pricing equation shows that annuity prices depend on the rates of return and mortality probabilities. Since changes in mortality probabilities occur very gradually, the equation implies that in the short run, changes in annuity prices can be explained exclusively by movements in the rate of return.

[7] Additional references of interest include Campbell and Viciera (2002), Cocco et al. (2005), Feldstein and Ranguelova (2001), Finkelstein et al. (2009), Lee and Carter (1992), Philips and Becker (1998), Poterba et al. (1997), and Richard (1975).

[8] This equation is based on what is known in the literature as the equivalence principal (see, e.g., Dickson, Hardy, and Waters, 2009; Promislow, 2011).

[9] This assumption is made for ease of exposition. We also use a more general equation for annuity price where interest rates are based on the term structure. We then derive the duration measure based on this more general equation. Under reasonable assumptions on possible changes in the term structure, the resulting duration values are generally very close to those obtained under this assumption results available upon request from the authors. See appendix 1 as well.

To obtain a measure for the sensitivity of annuity price to changes in interest rates, we will use the concept of (modified) duration, which is commonly used in the fixed-income literature to measure how bond prices adjust to changes in yields. Following the convention in that literature, we define the duration, D, of a life annuity as:

$$D = -\frac{1}{a(x,g,r)} \cdot \frac{\partial a(x,g,r)}{\partial r}. \tag{2}$$

The exact expression for D will depend on the expression for $a(x,g,r)$, which, in turn, depends on the assumed mortality law (which determines survival probabilities). We will use the Gompertz law of mortality, which has been shown to provide a reasonably accurate description of human mortality, particularly for individuals who are between middle age and early old age (e.g., Horiuchi and Coale, 1982; Haybittle, 1998).

Given Equations (1) and (2) and the Gompertz assumption, we derive the exact expression for the duration of a life annuity.[10] We then use the expression to calculate theoretical (or benchmark) duration values after we specify the rate of return and the mortality parameters. For the rate of return, we need a proxy that reflects the risk of insurance companies' investment portfolios. Insurance companies typically invest in several types of securities with various degrees of risk. According to a report by the National Association of Insurance commissioners (NAIC),[11] the four largest types of assets held by an average life insurance company as of July 2011 are corporate bonds (43.6 percent), U.S. government bonds (18.7 percent), structured securities such as mortgage-backed and asset-backed bonds (18.5 percent) and commercial mortgage loans (8.5 percent). Accordingly, one possible proxy is a weighted average of the rates of return on these four investment types, where the weights are based on the relative sizes of these four investment types. However, since the figures in the NAIC report came from a wide range of life insurance companies, including those that do not do much business in annuities, we believe that a better approach is to use two proxies to account for the least and the most risky of the four investment types (i.e., government bonds and mortgage-backed bonds, respectively). It is not uncommon for annuity studies to use more than one proxy for rates of returns. For example, Mitchell et al. (1999) use both Treasury rates and corporate bond yields in their calculations of money's worth of annuities.

We use the 10-year swap interest rate as a proxy for government bond yields. While swap rates are not entirely default-free (as they reflect the borrowing rates among large financial institutions), they are widely considered to be better proxies for risk-free rates than Treasury rates (Hull, Predescu, and White, 2005). For the yields on mortgage-backed securities, we use the 30-year mortgage interest rates. It is well known that mortgage rates are determined primarily by the rates of return on mortgage-backed securities (e.g., Roth, 1988). The two proxies will produce two different benchmark duration values, which we can interpret as the upper bound and the lower bound for duration values. If the empirical durations that we will shortly estimate lie outside

[10]The details of the derivation are available from the authors upon request.

[11]The information can be found at http://www.naic.org/capital_markets_archive/110819.htm.

TABLE 1
Theoretical Annuity Factors ($) and Durations, Years Calibrated to the Average of the 10-Year Swap Rates During the 2004–2012 Period

	Panel A: Annuity Factor, Male					Panel B: Annuity Factor, Female				
	Guarantee Period, Years					Guarantee Period, Years				
Age	20	15	10	5	0	20	15	10	5	0
55	17.02	16.61	16.33	16.17	16.12	18.00	17.80	17.67	17.60	17.58
60	16.04	15.42	14.98	14.73	14.64	16.92	16.57	16.35	16.23	16.19
65	15.16	14.23	13.56	13.16	13.03	15.85	15.27	14.89	14.68	14.61
70	14.45	13.15	12.15	11.53	11.32	14.91	13.97	13.33	12.97	12.86
75	13.99	12.27	10.84	9.89	9.56	14.21	12.81	11.78	11.17	10.97
80	13.77	11.67	9.73	8.34	7.83	13.83	11.94	11.94	9.36	9.02
	Panel C: Duration, Male					Panel D: Duration, Female				
	Guarantee Period, Years					Guarantee Period, Years				
Age	20	15	10	5	0	20	15	10	5	0
55	12.28	12.15	12.14	12.19	12.21	13.29	13.24	13.24	13.26	13.27
60	11.20	10.94	10.89	10.94	10.98	12.09	11.97	11.96	11.99	12.01
65	10.22	9.74	9.60	9.66	9.72	10.93	10.68	10.62	10.66	10.70
70	9.46	8.65	8.33	8.35	8.45	9.92	9.41	9.25	9.29	9.34
75	8.97	7.76	7.12	7.06	7.19	9.19	8.27	7.88	7.89	7.97
80	8.74	7.16	6.08	5.81	5.98	8.80	7.41	6.62	6.50	6.62

Note: Panels A and B display the theoretical *annuity factors* for male and female annuitants, respectively, of various ages and with various guarantee periods. Panels C and D display the theoretical *durations* of life annuities for male and female annuitants, respectively, of various ages and with various guarantee periods.

these bounds, we can conclude that real-life annuity prices do not promptly and proportionately adjust to interest rate movements.

For the mortality parameters, we use the following parameter values for the Gompertz distribution: $m = 88.18$ years and $b = 10.50$ years for males; and $m = 92.63$ years and $b = 8.78$ years for females. These values are taken from Milevsky and Young (2007a), who calibrate the Gompertz distribution to the Individual Annuity Mortality (IAM) 2000 table, updated with projection scale G. The IAM 2000 table was published by the Society of Actuaries.[12] It reflects the mortality statistics of people who buy life annuities (and who tend to have longer lifespans than that of the average population). The higher m value for females reflects the fact that women generally live longer than men do.

Tables 1 and 2 present the model prices and benchmark durations of single-life immediate annuities for various combinations of ages of annuitants (55–80 years old

[12]The IAM2000 is a baseline table which is then updated to a particular year using the projection scale G, which is the standard approach used in the actuarial literature.

TABLE 2
Theoretical Annuity Factors ($) and Durations, Years Calibrated to the Average of the 30-Year Mortgage Rates During the 2004–2012 Period

	Panel A: Annuity Factor, Male					Panel B: Annuity Factor, Female				
	Guarantee Period, Years					Guarantee Period, Years				
Age	20	15	10	5	0	20	15	10	5	0
55	14.45	14.13	13.89	13.75	13.70	15.09	14.93	14.82	14.76	14.74
60	13.80	13.31	12.95	12.72	12.64	14.40	14.12	13.94	13.83	13.79
65	13.20	12.48	11.92	11.56	11.44	13.68	13.23	12.91	12.72	12.66
70	12.71	11.69	10.86	10.30	10.10	13.03	12.30	11.77	11.44	11.33
75	12.38	11.04	9.84	8.99	8.67	12.54	11.45	10.58	10.03	9.8
80	12.22	10.58	8.95	7.71	7.22	12.27	10.79	9.47	8.57	8.24

	Panel C: Duration, Male					Panel D: Duration, Female				
	Guarantee Period, Years					Guarantee Period, Years				
Age	20	15	10	5	0	20	15	10	5	0
55	11.14	11.00	10.97	11.00	11.03	11.95	11.89	11.88	11.90	11.91
60	10.28	10.02	9.94	9.98	10.02	11.01	10.88	10.86	10.88	10.90
65	9.50	9.03	8.87	8.90	8.96	10.09	9.83	9.76	9.78	9.82
70	8.88	8.12	7.78	7.78	7.87	9.26	8.77	8.59	8.61	8.66
75	8.47	7.37	6.73	6.64	6.76	8.66	7.81	7.41	7.39	7.47
80	8.28	6.85	5.82	5.52	5.67	8.33	7.07	6.30	6.15	6.26

Note: Panels A and B display the theoretical *annuity factors* for male and female annuitants, respectively, of various ages and with various guarantee periods. Panels C and D display the theoretical *durations* of life annuities for male and female annuitants, respectively, of various ages and with various guarantee periods.

in increments of 5 years) and lengths of guarantee periods (0–20 years in increments of 5 years). In Table 1, the average of the 10-year swap rates between September 2004 and May 2012 (4.0641 percent p.a.; see Table 4) is used as the proxy for the rate of return, while in Table 2, the average of the 30-year mortgage rates over the same period (5.4648 percent p.a.; see Table 4) is used. In each table, Panels A and B display the fair annuity prices for male and female annuitants, respectively, while Panels C and D display the duration values for male and female annuitants, respectively.

Consider first Table 1. There are a few facts to note from the table. First, both the annuity prices and duration values are higher for female annuitants than for male annuitants. That is, a life annuity held by a woman is more expensive and more sensitive to interest rate changes (i.e., have a longer duration) than a life annuity held by a man of the same age. This is due to lower mortality rates of women, which translates to a longer expected stream of payments. Second, everything else being equal, the older the annuitant, the less expensive and less sensitive to interest rate movements the annuities are. This is consistent with the fact that mortality probabilities increase with age. Third, the length of the guarantee period has a positive effect on annuity prices, but a mixed effect on the duration values. Duration first declines as the length of the guarantee period increases but, after a certain length, starts to increase. To see why the effect on prices is positive, note from Equation (1) that a life annuity can be thought of as a portfolio of a guarantee component and a deferred-life-annuity component. A longer guarantee period increases the value of the guarantee component, but lowers the value of the life-contingent component. However, the increase in the former always more than offsets the decline in the latter. This is because under the Gompertz law, the force of mortality increases at a faster rate as one ages, which is consistent with empirical observations (until age 100 or so). This is why the effect of the guarantee periods on annuity prices is particularly strong for older annuitants, whose survival probabilities are progressively smaller.

To see why the effect on duration values is mixed, note that the duration of a life annuity is the weighted average of the durations of the guarantee component and the life-contingent component.[13] It can be shown that both of these durations increase with the guarantee period. However, their rates of increase are such that when they are weighted averaged (note that the weights also change as the values of the two components change), the resulting annuity duration has an ambiguous relationship with the length of the guarantee period.

Next, consider Table 2 where we assume that the rate of return on insurance companies' investments is equal to the average of the 30-year mortgage rates. Since this rate is higher than the 10-year swap rates used in Table 1 (which means that annuity providers earn more from investing annuity premiums), the fair annuity prices are lower. The same is true for duration values. In other words, higher rates of return reduce annuity prices and their sensitivity to changes in the rates.

Data Description and Summary Statistics

The U.S. annuity markets offer a wide variety of annuity products for both people who are in their working years (i.e., the accumulation phase) and people who are in their retirement (i.e., the payout phase). These products provide income that is fixed or variable (i.e., linked to an equity index). The income may be provided for a certain term (e.g., 10 years), for life, or for life with a guarantee period. Life annuities can also be based on one life or two lives (i.e., jointly with a spouse).

It can be challenging to quantify the size of the U.S. annuity markets. This is because many products that are sold as annuities may not lead to streams of annuity income payments. For example, an individual who purchases an annuity in her working years can choose to receive the payoffs at retirement in one lump sum, rather than conventional annuity payments. According to LIMRA (previously known as Life Insurance Marketing and Research Association), the total sales figure for fixed immediate life annuities (which provide fixed income payments for life starting immediately, the sub-

[13]This is similar to the fact that the duration of a bond portfolio is the weighted average of the durations of the individual bonds in the portfolio.

ject of this article) was $7.6 billion in 2010, and the total sale of all annuity products in the same year was $221 billion.

Our data set consists of weekly annuity quotes for single-premium immediate annuities that were offered in the United States during the period from September 2004 to May 2012.[14] The quotes are in terms of annuity payouts (i.e., amounts of monthly income) per $100,000 of annuity principal.[15] The annuities are classified along the following five dimensions: ages of annuitants (55–80 years old in increments of 5 years), gender (male and female), guarantee periods (0 to 25 years in increments of 5 years), payout options (single life, joint life, and term certain), and income tax treatment of the annuity income (qualified or nonqualified).[16]

Every week, the incoming quotes were validated and checked for irregularities. In total, the data set contains over 3 million quotes from 25 life insurers (not all of them offered every combination of age and guarantee period).

For the purposes of this article, we focus on qualified single-premium, single-life annuities. These annuities are purchased with savings inside a retirement savings account (e.g., a DC pension plan or an IRA). Thus, all annuity income would be treated as ordinary interest and we would not be at risk of observing tax effects. Out of all qualified annuities in the data set, we limit our attention to six age groups (55, 60, 65, 70, 75, and 80 years old) and five guarantee periods (0, 5, 10, 15, and 20 years). Therefore, our sample has in total 30 different annuity combinations for male annuitants, and another 30 combinations for female annuitants. We then exclude the following combinations (for both male and female) due to the small number of firms providing quotes for them: age 75 with a 20-year guarantee, age 80 with a 20-year guarantee, and age 80 with a 15-year guarantee. This leaves us with 54 combinations. We believe that these 54 combinations are sufficiently representative of all qualified annuities in our data set.

For every annuity combination, we use the following three steps to transform the quoted payouts from various providers into annuity prices. First, for every week in the sample period, we average the quoted payouts after we remove the highest and the lowest quotes (in order to limit the effects of outliers).[17] Second, we annualize the payouts by multiplying them by 12. Finally, we divide the annualized payouts into $100,000 to get the annuity prices per $1 of payout per year (henceforth alternately

[14]The process through which we obtained the data is described in Appendix B.

[15]For example, the quote of 650 means that if one buys $100,000 worth of this life annuity, one would get monthly income of $650 for life.

[16]A qualified annuity is purchased as part of a retirement plan that is set up by an employer (e.g., a DC pension plan) or by the annuitant (e.g., an Individual Retirement Account [IRA]). Contributions made to qualified annuities are typically deductible from the gross income of the individual for income tax purposes. On the other hand, a nonqualified annuity is purchased with after-tax dollars. As a result, the tax treatment of annuity payouts will be different between the two types.

[17]Weeks that do not have enough quotes to generate reliable averages are removed from the data set.

referred to as "annuity prices" or "annuity factors"). These three steps provide us with a time series of weekly annuity factors for each annuity combination.

The summary statistics are presented in Table 3 (Panel A for male and Panel B for female). The empirical annuity factors display similar properties to those of the benchmark factors in Tables 1 and 2, namely: (1) the factors are higher for women than for men; (2) the older the annuitant, the lower the factors are; and (3) the longer the guarantee period, the higher are the factors. Also, we note that the empirical annuity factors typically lie between the model prices displayed in Table 1 (calibrated to the 10-year swap rate) and Table 2 (calibrated to the 30-year mortgage rate), and typically are nearer to the former.

To obtain more intuition on the annuity factors in Table 3, consider, for example, the annuity for a 60-year-old male with no guarantee period. The annuity factor is \$14.564. This is the price that he has to pay in exchange for a lifetime income of \$1 per year. Suppose that he buys \$100,000 worth of this annuity. He will then receive $\$100,000/14.564 = \$6,866.25$ per year (or about \$572 per month) for life. Alternatively, we can present the same information in terms of annuity rates, which are the inverse of annuity factors. That is, annuity rates are the rates of income on the annuities. In this case, the annuity rate is 6.87 percent p.a.

At any given time, annuity rates are higher than prevailing bond yields (of similar credit risk). This is because of mortality credits or risk pooling under which those who die early subsidize those who live longer. Figure 1 displays the time series of annuity rates over the sample period for selected annuities (i.e., men aged 60, 65, and 70 with no guarantee period in Panel A, women in Panel B). For comparison, the 10-year swap interest rates are also plotted. As can be seen, the three series of annuity rates move together, which is to be expected because in the short run, changes in annuity prices/rates should depend mainly on the movements in interest rates. The annuity rates are higher for men than for women, reflecting men's shorter life expectancies. The annuity rates were trending up slightly between 2004 and the end of 2008, and declined afterward. The 10-year swap interest rates have a similar up-and-then-down trend. However, they appear to have much more variability. That is, annuity rates do not appear to fluctuate as much as the 10-year swap rates do.

SENSITIVITY ESTIMATIONS

To measure the sensitivity of observed annuity factors to changes in interest rates, we estimate the durations of the annuities in our sample. This is done by regressing the *negative* of the percentage changes in the annuity factors over a specified period of time on the changes in interest rates over the same time period. The resulting slope coefficients will be the estimated sensitivity (i.e., empirical durations).[18]

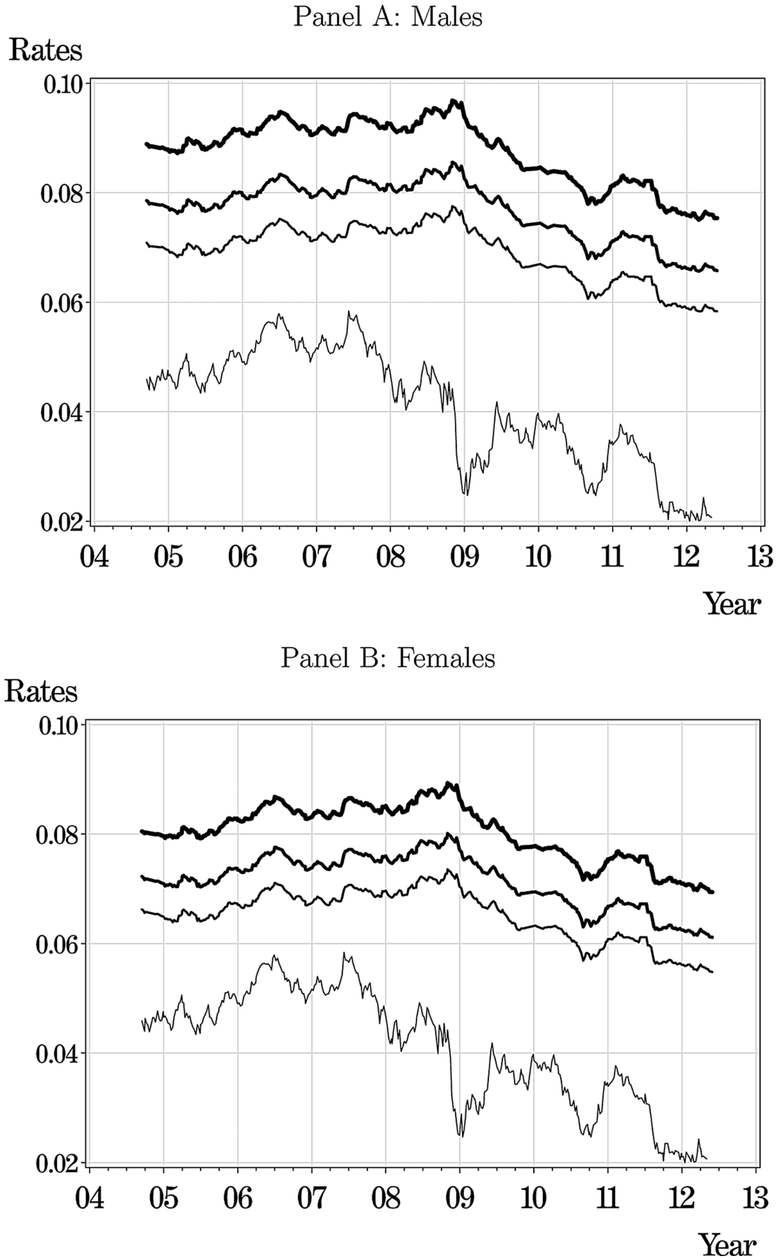
[18]Based on Equation (2), the negative of the percentage change in the annuity factor is approximately equal to the change in interest rate multiplied by the duration. See Equation (6) below.

TABLE 3
Summary Statistics on the Empirical Annuity Factors

	Guarantee Period, Years				
Age	20	15	10	5	0
Panel A: Summary Statistics on the Empirical Annuity Factors, Male Annuitant					
55	16.60	16.26	16.02	15.86	15.82
	(1.37)	(1.34)	(1.33)	(1.35)	(1.34)
	[14.6, 19.8]	[14.3, 19.3]	[14.1, 19.1]	[14.0, 18.9]	[13.9, 18.8]
60	15.79	15.25	14.87	14.63	14.56
	(1.24)	(1.19)	(1.17)	(1.18)	(1.17)
	[14.0, 18.6]	[13.5, 18.0]	[13.2, 17.5]	[13.0, 17.3]	[12.9, 17.1]
65	15.01	14.19	13.58	13.20	13.08
	(1.11)	(1.02)	(0.99)	(0.99)	(0.99)
	[13.4, 17.5]	[12.7, 16.5]	[12.2, 15.8]	[11.8, 15.4]	[11.7, 15.2]
70	14.38	13.22	12.28	11.67	11.47
	(1.05)	(0.88)	(0.84)	(0.84)	(0.84)
	[13.0, 16.7]	[11.9, 15.2]	[11.1, 14.1]	[10.5, 13.5]	[10.3, 13.3]
75		12.41	11.08	10.12	9.80
	N/A	(0.81)	(0.71)	(0.71)	(0.72)
		[11.2, 14.2]	[10.1, 12.5]	[9.2, 11.6]	[8.9, 11.3]
80			10.02	8.64	8.12
	N/A	N/A	(0.66)	(0.59)	(0.63)
			[9.2, 11.3]	[7.9, 9.8]	[7.4, 9.4]
Panel B: Summary Statistics on the Empirical Annuity Factors, Female Annuitant					
55	17.09	16.87	16.71	16.61	16.59
	(1.43)	(1.40)	(1.39)	(1.40)	(1.39)
	[15.0, 20.5]	[14.8, 20.2]	[14.7, 20.0]	[14.6, 19.9]	[14.5, 19.8]
60	16.24	15.88	15.63	15.48	15.44
	(1.29)	(1.24)	(1.22)	(1.23)	(1.22)
	[14.3, 19.3]	[14.0, 18.8]	[13.8, 18.5]	[13.7, 18.4]	[13.6, 18.2]
65	15.36	14.77	14.37	14.13	14.06
	(1.14)	(1.07)	(1.04)	(1.03)	(1.03)
	[13.7, 18.0]	[13.2, 17.2]	[12.8, 16.7]	[12.6, 16.5]	[12.5, 16.4]
70	14.59	13.68	13.02	12.61	12.50
	(1.07)	(0.92)	(0.87)	(0.86)	(0.86)
	[13.2, 17.0]	[12.3, 15.8]	[11.7, 15.0]	[11.3, 14.6]	[11.2, 14.4]
75		12.68	11.63	10.95	10.74
	N/A	(0.82)	(0.73)	(0.71)	(0.72)
		[11.4, 14.5]	[10.6, 13.2]	[9.9, 12.4]	[9.7, 12.2]
80			10.37	9.26	8.88
	N/A	N/A	(0.66)	(0.59)	(0.61)
			[9.5, 11.7]	[8.4, 10.5]	[8.0, 10.2]

Note: This table displays the summary statistics on empirical annuity factors for male (Panel A) and female (Panel B) annuitants of various ages and with various guarantee periods, over the period 2004–2012. Standard errors are in parentheses. Minimum and maximum are in brackets.

FIGURE 1
10-Year Swap Rates Versus Annuity Payout Rates



Note: Plot of annuity rates and 10 year swap rates across age groups for males and females, 0-year guarantee. The 10-year swap rate is the lowest line and the annuity rates are plotted to increase in thickness as age extends, ages 60, 65, 70.

Let $a_i(x,g)$ be the annuity factor observed in week i for an x-year-old individual with a guarantee period of g years. The percentage change in the value of this factor over a k-week period (i.e., from week $(i-k)$ to week i, $k \geq 1$) is denoted by:

$$\frac{\Delta a_i(x,g|k)}{a_{i-k}(x,g)} = \frac{a_i(x,g) - a_{i-k}(x,g)}{a_{i-k}(x,g)}. \tag{3}$$

Let $r_i(T)$ denote the interest rate for the term of T years observed in week i. The change in the rate from week $(i-k)$ to week i, is denoted by:

$$\Delta r_i(T|k) = r_i(T) - r_{i-k}(T). \tag{4}$$

Our regression equation is:

$$-\frac{\Delta a_i(x,g|k)}{a_{i-k}(x,g)} = \beta_0^{x,g} + \beta_{\Delta r_i(T|k)}^{x,g} \Delta r_i(T|k) + \boldsymbol{\varepsilon}_i. \tag{5}$$

If annuity factors adjust promptly and fully to the changes in interest rates, $\beta_0^{x,g}$ should be equal to zero and $\beta_{\Delta r_i(T|k)}^{x,g}$ should be close to the benchmark duration values in Table 1 or Table 2, depending on the choice of proxy for interest rates.

Table 4, Panel A displays the summary statistics of the 10-year swap rates, the 30-year mortgage rates, and the weekly changes in both rates. The average of the 10-year swap rates during the sample period is 4.0641 percent p.a., while the average of their weekly changes is −0.0068 percent p.a., with a standard deviation of 0.1357 percent p.a. The average absolute magnitude of the changes during the sample period is 0.1016 percent p.a. (not shown in the table). That is, on average, the magnitude of weekly changes in the 10-year swap interest rates is about 10 basis points. Out of the 403 weeks in our sample period, 164 of them experienced a change of at least 0.10 percent p.a. in either direction. In addition, 29 of the weeks experienced a change of at least 0.25 percent p.a. in absolute value.

Panels B and C of Table 4 display the summary statistics of the weekly changes in the dependent variable (expressed in percentage terms). As defined in Equation (5), the dependent variable is the *negative* of the rate of change in annuity factors. During the sample period, the average weekly changes in the dependent variable are typically between −0.02 percent and −0.05 percent.[19] For older annuitants (i.e., age 65 and up), the weekly changes are smaller and less volatile, as evidenced by lower standard deviations and narrower ranges between the minimum and the maximum values.

For all annuitant ages, large changes in the annuity factors did not occur as frequently as changes in the 10-year swap rates would suggest. To see this, we will use the 10 annuity combinations for a 60-year-old person (i.e., five guarantee periods each for

[19] In other words, the average weekly changes in the annuity factors are typically between +0.02 percent and +0.05 percent, which is consistent with the fact that during our sample period, annuity rates were declining (see Figure 1) and thus annuity prices were increasing.

Table 4
Summary Statistics on Relevant Interest Rates and on Weekly Percentrage Changes in Empirical Annuity Factors

Panel A: Summary Statistics on Relevant Interest Rates

Variable (%)	N	Mean	Std	Min	Max	Skew	Kurt
10-year swap rate	403	4.0641	1.0681	1.87	5.84	−0.36	−0.99
Δ10-year rate: $\Delta r_i(10\|1)$	402	−0.0068	0.1357	−0.71	0.52	−0.06	2.32
30-year mortgage rate	402	5.4648	0.8483	3.75	6.80	−0.35	−1.10
$\Delta 30$ − year mortgage rate $\Delta r_i(30\|1)$	400	−0.0047	0.0992	−0.44	0.52	0.50	5.75

	Guarantee Period, Years				
Age	20	15	10	5	0
Panel B: Summary Statistics on Weekly Percentage Changes in Empirical Annuity Factors, Male Annuitant					
55	−0.052	−0.050	−0.047	−0.047	−0.045
	(0.613)	(0.612)	(0.607)	(0.625)	(0.606)
	[−2.49, 1.61]	[−2.47, 1.98]	[−2.45, 1.59]	[−2.47, 2.00]	[−2.49, 1.60]
60	−0.047	−0.044	−0.042	−0.041	−0.040
	(0.567)	(0.555)	(0.554)	(0.568)	(0.551)
	[−2.30, 1.52]	[−2.28, 1.48]	[−2.35, 1.46]	[−2.37, 1.82]	[−2.36, 1.45]
65	−0.036	−0.037	−0.035	−0.034	−0.032
	(0.583)	(0.501)	(0.499)	(0.509)	(0.497)
	[−2.71, 2.89]	[−2.04, 1.40]	[−2.04, 1.39]	[−2.08, 1.60]	[−2.09, 1.28]
70	−0.040	−0.033	−0.030	−0.029	−0.028
	(0.636)	(0.451)	(0.457)	(0.464)	(0.452)
	[−2.27, 3.32]	[−1.73, 1.19]	[−1.86, 1.18]	[−1.86, 1.57]	[−1.85, 1.18]
75		−0.029	−0.021	−0.022	−0.021
	N/A	(0.555)	(0.420)	(0.433)	(0.413)
		[−2.11, 4.04]	[−1.76, 1.15]	[−1.81, 1.60]	[−1.81, 1.36]
80			−0.026	−0.025	−0.026
	N/A	N/A	(0.487)	(0.392)	(0.399)
			[−1.89, 3.52]	[−1.98, 1.46]	[−1.98, 1.58]
Panel C: Summary Statistics on Weekly Percentage Changes in Empirical Annuity Factors, Female Annuitant					
55	−0.054	−0.052	−0.051	−0.051	−0.049
	(0.649)	(0.649)	(0.651)	(0.672)	(0.643)
	[−2.72, 1.79]	[−2.76, 1.78]	[−2.85, 1.81]	[−2.71, 2.36]	[−2.75, 1.78]
60	−0.048	−0.046	−0.044	−0.044	−0.042
	(0.599)	(0.596)	(0.598)	(0.614)	(0.596)
	[−2.44, 1.55]	[−2.42, 1.52]	[−2.53, 1.53]	[−2.46, 2.15]	[−2.46, 1.55]
65	−0.037	−0.037	−0.035	−0.034	−0.033
	(0.621)	(0.538)	(0.539)	(0.549)	(0.536)
	[−3.05, 3.12]	[−2.22, 1.45]	[−2.24, 1.46]	[−2.28, 1.82]	[−2.29, 1.36]
70	−0.039	−0.034	−0.031	−0.031	−0.029
	(0.656)	(0.478)	(0.485)	(0.489)	(0.481)
	[−2.40, 3.35]	[−1.86, 1.28]	[−1.87, 1.26]	[−1.88, 1.25]	[−1.90, 1.27]
75		−0.026	−0.020	−0.022	−0.023
	N/A	(0.568)	(0.435)	(0.444)	(0.436)
		[−2.25, 3.92]	[−1.82, 1.11]	[−1.73, 1.13]	[−1.69, 1.14]
80			−0.022	−0.023	−0.025
	N/A	N/A	(0.484)	(0.401)	(0.407)
			[−2.02, 3.50]	[−1.97, 1.14]	[−1.65, 1.20]

Note: Panel A displays the summary statistics for weekly levels and changes in the 10-year swap interest rate and 30-year conventional mortgage rate, over 2004–2012, measured in percentages. The mortgage rate is sourced from the Board of Governors of the Federal Reserve System, series id WRMORTG. Panel B displays the summary statistics of weekly changes in annuity factors, male annuitants. Panel C displays the summary statistics of weekly changes in annuity factors, female annuitants. Standard errors are in parentheses. Minimum and maximum are in brackets.

male and female) as examples. Recall that Equation (2) approximates the changes in the annuity factor as:

$$-\frac{\Delta a(x,g,r)}{a(x,g,r)} \approx D \cdot \Delta r, \tag{6}$$

where the equation is rearranged and presented in a discrete form. As the benchmark durations of annuities for a 60-year-old person is between 10 and 12 years (see Panels C and D of Tables 1 and 2), a 0.10 percent change in interest rates should translate to a change in the annuity factor of approximately 1 percent to 1.20 percent. During the sample period, none of the 10 annuities had a change of 1 percent or greater more than 33 times. This is despite the fact that, as stated earlier, a change of 10 basis points or larger in the 10-year swap rates occurred over 160 times during the same period.[20] This is another indication that annuity factors are not adjusted as immediately or fully to changes in interest rates.

Regressions Using the 10-Year Swap Rates

We start our empirical tests by running the regression in Equation (5) for every annuity combination in our sample, using the 10-year swap rates as the proxy for interest rates. Changes in the annuity factors and changes in the swap rates are measured over a period of *1 week* (i.e., measurement period $k = 1$). The duration estimates for all annuity combinations (not shown, but available upon request) are well below and statistically different from the benchmark durations in Table 1. Several of the estimates are less than 1, particularly for older annuitants (70 years old and higher). The highest empirical duration of all combinations is only 1.46. This suggests that real-life annuity factors are far from being responsive to the weekly movements in the 10-year swap interest rates. The magnitude of the adjustments from week to week are only a small fraction (about 1/10th) of the benchmark values. The R^2's of the regressions are all very low, most of which are around 7 percent to 8 percent, indicating that weekly movements in the 10-year swap rates have very low explanatory power.

The results confirm that insurance companies do not change their annuity quotes (prices) from week to week to correspond fully to changes in risk-free interest rates during that week. It is possible (in fact, likely) that insurance companies take some time before they change their prices. To verify this, we look at the adjustments over a longer period than 1 week.[21] We repeat the regressions in the previous subsection with the independent variable ($\Delta r_i(T|k)$) and dependent variable measured over periods of 2 weeks up to 20 weeks (i.e., $2 \leq k \leq 20$). In these regressions, we use overlapping

[20] Larger changes in annuity prices are even more rare. Of the 10 annuities for a 60-year-old person, only one had a change of larger than 2.50 percent (and only for one time). This is despite the fact that a 25-basis-point (or more) change in the 10-year swap rates happened 29 times during the sample period.

[21] A longer period also reduces any problems of non-synchronicity in the data. Annuity quotes are issued during the Wednesday of the week and are a firm commitment for a period of several days, while our Wednesday swap rates are obtained from secondary market rates at day's close on Wednesday.

TABLE 5
Estimated Durations: Age 65 With No Guarantee Period Measurement Period $k = 1, \ldots, 20$ Weeks, Using 10-Year Swap Rates

	Male			Female		
k	N	$\beta^{65,0}_{\Delta r_i(10\|k)}$	R^2	N	$\beta^{65,0}_{\Delta r_i(10\|k)}$	R^2
1	345	**0.94** (0.24)	0.06	347	**1.05** (0.23)	0.07
2	342	**1.76** (0.30)	0.19	344	**1.88** (0.31)	0.19
3	341	**2.33** (0.41)	0.29	343	**2.42** (0.43)	0.27
4	339	**2.58** (0.51)	0.32	343	**2.71** (0.54)	0.31
5	337	**2.75** (0.57)	0.35	340	**2.91** (0.60)	0.35
10	332	**2.79** (0.81)	0.33	334	**2.93** (0.81)	0.34
15	329	**3.03** (0.94)	0.34	331	**3.11** (0.94)	0.34
20	321	**3.04** (0.94)	0.34	324	**3.16** (0.95)	0.34

Note: This table displays the estimated durations of life annuities for male and female annuitants, age 65 and 0 year guarantee. The independent variable of the regressions is the lagged weekly change in the 10-year swap rates. The dependent variable is the negative of the weekly percentage changes in the annuity factors. All regressions are performed one at a time, estimated with GMM and HAC standard errors. To calculate the HAC standard errors we follow Newey and West (1994). Bolded coefficients denote significance at the 0.05 level, based on two-sided tests, of a difference between the estimated Duration coefficient and the theoretical value. Standard errors are in parentheses.

weekly observations.[22] The standard errors of the estimates are calculated using the Newey–West procedure, which accounts for this overlap and the induced moving average error term.

We compare the estimated durations and the R^2's of those regressions across all ages, guarantee periods, and measurement periods. Since there are numerous regression combinations and their results tend to be similar, we will only present the results for annuities for a 65-year-old male and a 65-year-old female with no guarantee period, which is the age typically associated with retirement. The regression estimates are displayed in Table 5.

[22]For example, for the measurement period of $k = 4$ weeks, the first observation is the change from week 1 to week 5, while the second observation is the change from week 2 to week 6, and so on.

As can be seen from the table, both the estimated durations and the R^2's are higher when changes are measured over longer periods. The estimated durations increase rapidly as the measurement period lengthens from 1 week ($k = 1$) to 5 weeks ($k = 5$). They then gradually increase to reach a plateau value of slightly over 3 years. While these duration values are higher than in the case where $k = 1$ week, they are still far below (and significantly different from) the benchmark values in Panels C and D of Table 1 (9.72 years for male and 10.70 years for female). The R^2's reach their peak at around 35 percent when $k = 5$, and remain at approximately the same level beyond that length.

The fact that the estimated sensitivity and the R^2's increase when measured over longer periods supports the notion that insurance companies do not instantaneously adjust the annuity factors to respond to every change in interest rates (as proxied by the swap rates). Rather, the adjustments are gradual and can take several weeks. Even over a long period, the adjustments are far from complete. The magnitude of the changes in annuity prices is much smaller than the benchmark levels.[23]

Regressions Using the 30-Year Mortgage Rates

The sluggish and partial response that we observe may be due to our use of the 10-year swap rates as the proxy for interest rates. In other words, the swap rates (which are close to the risk-free rates) may not be representative of the rates of return that annuity providers earn on their investments. As a result, changes in the swap rates may not have much influence on how annuity providers set their prices. To test this conjecture, we now change the proxy for interest rates to be the 30-year mortgage rates.[24] These rates are obtained from the Federal Home Loan Mortgage Corporation's Primary Mortgage Market Survey, which is conducted every week. The survey collects information from various mortgage lenders on the rates for several kinds of mortgage products. The 30-year mortgage rates are the rates that borrowers can expect to be offered if they request 30-year fixed-rate mortgage loans on the survey day. Since the rates can vary across lenders, the number reported for each week is the weighted average of all the survey responses in that week, where the weights are based on the dollar volume of mortgage transactions.[25]

[23]To account for the possibility that a linear regression may not capture the convexity in the relationship between interest rates and annuity factors, especially at longer lags, we also perform the above regressions with the squares of changes in the swap rates as an additional independent variable. The estimated coefficients for the squares of changes are not significantly different from zero for all lags (except one), suggesting that nonlinearity is not a concern. The estimated durations are only slightly higher than before (i.e., by no more than 0.50 years regardless of the measurement period). These duration values are again still far short of the benchmark durations. Similarly, the improvement in R^2's is minimal (higher by 2 percent to 3 percent). Accordingly, we conclude that the low estimated sensitivity in Table 5 is not likely to be the result of a simple model misspecification involving nonlinearity.

[24]The mortgage rates were suggested to us by insurance practitioners as a possible proxy for their pricing basis and the use of these rates is also supported by the presence of mortgage-backed securities in the asset portfolios of the typical insurance firm.

[25]For more details about how the survey is conducted and how the averages are calculated, see the Web site of the survey at: http://www.freddiemac.com/pmms/abtpmms.htm.

Table 4, Panel A displays the summary statistics for the 30-year mortgage rates. The average of the rates over the sample period is 5.4648 percent p.a. The 30-year mortgage rates are less volatile than the 10-year swap rates. The average weekly change in the rates during the sample period is −0.0047 percent p.a., while the standard deviation is 0.0992 percent p.a., both of which values are lower than their counterparts for the 10-year swap rates. The range between the minimum and the maximum changes is also narrower. Although not shown, the average of the absolute values of the changes is 0.0676 percent p.a., which again is lower than the 10-year-swap-rate counterpart. The frequencies of large changes are also lower, with changes larger than 0.10 percent (0.25 percent) p.a. happening only in 83 (13) of the weeks. These observations are consistent with past findings that mortgage rates move more slowly than market interest rates (e.g., Allen, Rutherford, and Wiley, 1999; Payne, 2006).

As before, we run the regressions for all annuity combinations over various measurement periods. The regression equation is the same as in Equation (5) except that the independent variable is now changes in the 30-year mortgage rates.[26] Examination of the patterns of the estimated durations and R^2's show that they are similar across annuitants' ages. The representative patterns are shown in Figure 2 (for R^2's) and Figure 3 (for durations), using results from annuities for a 65-year-old person. The duration values are very low (around 1 year) when changes are measured over a period of 1 week. They then rapidly increase to about 4 years when the measurement period is 5 weeks, and over 5 years when the measurement period is 12 weeks and beyond. The R^2's follow a similar pattern, starting off at 10 percent when the measurement period is 1 week and then rising rapidly before stabilizing at around 50 percent when the measurement period is 16 weeks and beyond.

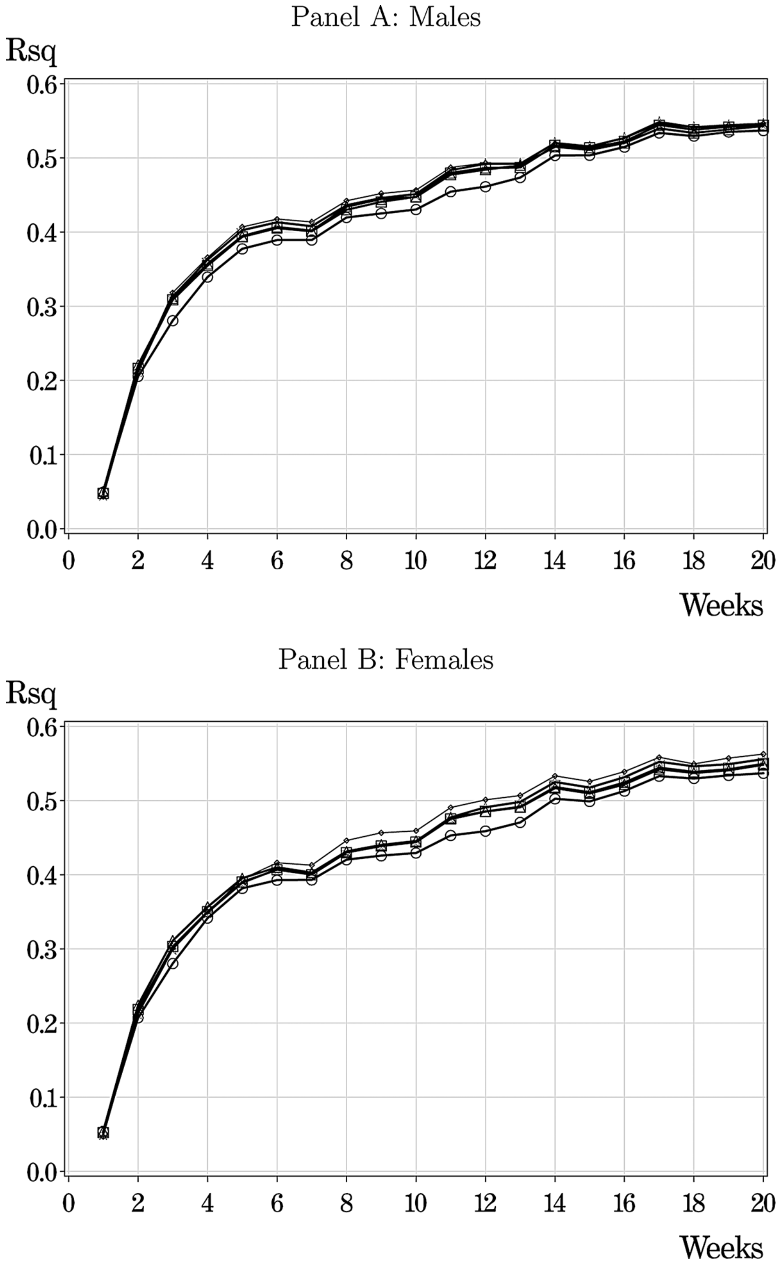
Since the estimated durations and R^2's remain approximately the same beyond $k = 16$ weeks, we present in Table 6 the results for this measurement period for all annuity combinations. Duration values (for both male and female) never exceed 6.50 years, and most of them are between 4.50 and 6.50 years. While these durations are much higher than what we observe when the 10-year swap rates are used, they are still significantly below the benchmark values in Table 2. As for the R^2's, most of them are in the 50 percent range, indicating a substantially improved fit with the use of mortgage rates compared to the 10-year swap rates.

Taken together, the results in Table 6 and Figures 2 and 3 suggest that annuity prices are more closely related to changes in the mortgage rates than to the swap rates. Nevertheless, the fact that the estimated durations are still significantly lower than the benchmark values, especially when the measurement period is short (e.g., $k = 1$) suggests that annuity prices are still not very responsive to movements in rates.

30-Year Mortgage Rates and Asymmetric Responses. Next, we want to account for the possibility that the manner in which insurance companies react to changes in interest rates can be different between a rate increase and a rate decrease. Studies on

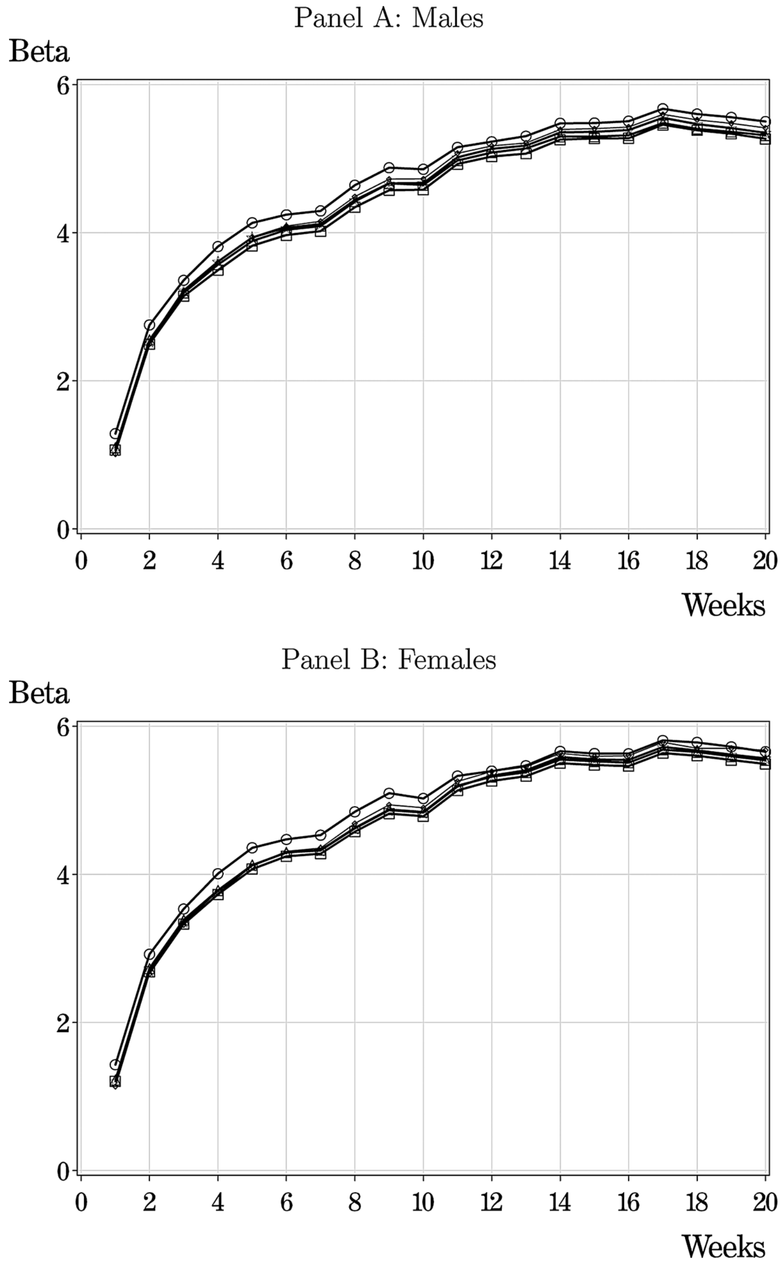
[26]As before, including the squares of changes in interest rates in this regression does not contribute meaningfully to the values of estimated durations or R^2's.

FIGURE 2
R^2 as a Function of the Observation Lag 30-Year Mortgage Rate



Note: Plot of R^2 for males and females, age 65, qualified, across guarantee periods: Guarantee Period = 0: $\diamond$ Guarantee Period = 5: $\star$ Guarantee Period = 10: $\square$ Guarantee Period = 15: $\triangle$ Guarantee Period = 20: $\circ$.

FIGURE 3
Estimated Duration as a Function of the Observation Lag 30-Year Mortgage Rate



Note: Plot of estimated duration coefficients for males and females, age 65, qualified, across guarantee periods: Guarantee Period = 0: ◇ Guarantee Period = 5: ⋆ Guarantee Period = 10: □ Guarantee Period = 15: △ Guarantee Period = 20: ○ .

TABLE 6
Estimated Durations: All Ages and Guarantee Periods Measurement Period $k = 16$ Week, Using 30-Year Mortgage Rates

	Panel A: Durations, Male					Panel B: Durations, Female				
	Guarantee Period, Years					Guarantee Period, Years				
Age	20	15	10	5	0	20	15	10	5	0
55	**6.29**	**6.21**	**6.29**	**6.40**	**6.37**	**6.42**	**6.37**	**6.37**	**6.45**	**6.46**
60	**5.97**	**5.82**	**5.83**	**5.92**	**5.90**	**6.04**	**5.95**	**5.96**	**6.03**	**6.05**
65	**5.50**	**5.31**	**5.27**	**5.38**	**5.43**	**5.63**	**5.51**	**5.46**	**5.54**	**5.60**
70	**4.59**	**4.84**	**4.85**	**4.98**	**5.10**	**4.59**	**5.01**	**5.00**	**5.10**	**5.22**
75	N/A	**3.91**	**4.27**	**4.37**	**4.57**	N/A	**3.93**	**4.41**	**4.48**	**4.65**
80	N/A	N/A	**3.15**	**3.80**	**4.08**	N/A	N/A	**3.28**	**3.96**	**4.15**
	Panel C: R^2s, Male					Panel D: R^2s, Female				
	Guarantee Period, Years					Guarantee Period, Years				
Age	20	15	10	5	0	20	15	10	5	0
55	0.512	0.509	0.517	0.524	0.523	0.511	0.512	0.514	0.518	0.526
60	0.519	0.516	0.519	0.522	0.520	0.515	0.516	0.520	0.525	0.531
65	0.514	0.520	0.522	0.527	0.526	0.513	0.522	0.524	0.531	0.539
70	0.445	0.515	0.527	0.537	0.542	0.439	0.519	0.528	0.542	0.552
75	N/A	0.421	0.510	0.510	0.518	N/A	0.419	0.520	0.528	0.541
80	N/A	N/A	0.384	0.459	0.441	N/A	N/A	0.395	0.481	0.467

Note: This table displays the estimated durations of life annuities, together with the model R^2s, derived from the parameter estimates of models of the form:

$$-\Delta a_i(x,g|16)/a_{i-k}(x,g) = \beta^{x,g} + \beta^{x,g}_{\Delta r_i(30|16)} * \Delta r_i(30|16) + \epsilon_i^{x,g}.$$

The independent variable of the regressions is the lagged change in the 30-year mortgage rates measured over a period of 16 weeks, $\Delta r_i(30|16)$. The dependent variable is the negative of the percentage changes in the annuity factors over the same measurement period for male annuitants of various ages and with various guarantee periods. All regressions are performed one at a time, estimated with GMM and HAC standard errors. To calculate the HAC standard errors we follow Newey and West (1994). Bolded coefficients denote significance at the 0.05 level, based on two-sided tests, of a difference between the estimated Duration coefficient and the theoretical value.

consumer interest rates such as bank deposit rates have shown that when interest rates (e.g., T-bill rates) change, U.S. banks are slower to raise deposit rates than to reduce them (e.g., Hannan and Berger, 1991; Neumark and Sharpe, 1992). We want to know whether such an asymmetric response also occurs in the annuity markets. To this end, we specify the regression equation as follows:

$$-\frac{\Delta a_i(x,g|k)}{a_{i-k}(x,g)} = \beta_k^{x,g} + \beta^{x,g}_{\Delta r_{i,Neg.Sign}(30|k)} \cdot \Delta r_i(30|k) \cdot I_{\Delta r,neg} + \beta^{x,g}_{\Delta r_i(30|k)} \cdot \Delta r_i(30|k) + \varepsilon_i, \tag{7}$$

where $a_i\,(x,g|k)$ is as previously defined, $\Delta r_i(30|k)$ is the change in the 30-year mortgage rates over the measurement period k, and $I_{\Delta r,neg}$ is an indicator variable that takes on the value of one when interest rates decline (i.e., $\Delta r_i(T|k) < 0$), and zero otherwise. Accordingly, $\beta^{x,g}_{\Delta r_{i,Neg.Sign}(30|k)} + \beta^{x,g}_{\Delta r_i(30|k)}$ is the estimated duration when $\Delta r_i(T|k) < 0$, and $\beta^{x,g}_{\Delta r_i(30|k)}$ is the estimated duration when $\Delta r_i(T|k) > 0$.

The regressions results are displayed in Table 7. As before, we choose to present the results for the measurement period of $k = 16$ weeks. Results for other measurement periods are available on request.

For every annuity combination (for both male and female), the estimated duration is much higher when interest rates increase (Panels A and B) than when interest rates decline (Panels C and D). When rates increase, the durations are generally about 5–9 years, which are slightly higher than the durations in the previous set of regressions (i.e., without asymmetric responses). However, these durations are still typically significantly different from the benchmark values in Table 2. On the other hand, when rates decline, the estimated durations are only about 3–5 years. These results suggest that annuity prices' responses to changes in the 30-year mortgage rates are asymmetric. When interest rates increase, insurance companies reduce annuity prices (by increasing the amounts of payouts) by a greater magnitude than what they do in the opposite direction when interest rates decline. This downward-sticky pattern of actions is beneficial to the customers. It also raises the possibility that potential annuitants can time their purchases based on the observed movements in interest rates.

Discussion

The observed sluggish and asymmetric responses likely reflect market imperfections ignored in standard annuity-pricing models. For example, insurance companies may want to smooth out the price changes and/or to wait in order to get a better sense of the trend of interest rates. Another possible explanation is that insurance companies from time to time have unbalanced books (i.e., mismatches of durations of assets and liabilities). These companies may deliberately offer uncompetitive annuity prices or delay price adjustments in order to allow themselves time to rebalance their books, resulting in the sluggish responses that we observe.

The downward-sticky response is consistent with the customer reaction hypothesis outlined in Rotemberg (1982). Adapted to our context, the hypothesis states that insurance companies, especially those operating in a highly competitive market, may fear a negative reaction from potential annuity buyers if annuity payout rates are reduced. Therefore, they will try to delay a rate reduction as long as they can. In the meantime, they absorb the loss through the profit margin that they build into the annuity prices. While the U.S. annuity markets are dominated by only a few large providers, the results of the money's worth studies indicate that these few providers do not abuse their oligopolistic power, as evidenced by the fact that the money's worth ratios are generally not far below unity (see Mitchell et al., 1999; James and Song, 2001).[27] That is, the U.S. annuity markets appear to operate in a competitive manner.

[27] This argument follows a similar one made by Cannon and Tonks (2005).

TABLE 7

Asymmetric Regression, Male and Female Annuitants Estimated Durations: All Ages and Guarantee Periods Measurement Period $k = 16$ Week Using 30-Year Mortgage Rates

	Panel A: Durations When Interest Rates Increase, Male					Panel B: Durations When Interest Rates Increase, Female				
	Guarantee Period, Years					Guarantee Period, Years				
Age	20	15	10	5	0	20	15	10	5	0
55	**8.48**	**8.32**	**8.42**	**8.61**	**8.39**	**9.00**	**8.90**	**8.86**	**9.07**	**8.81**
60	**8.11**	**7.89**	**7.89**	**8.01**	**7.84**	**8.53**	**8.35**	**8.31**	**8.47**	**8.26**
65	**7.40**	**7.12**	**6.98**	**7.07**	**6.99**	**7.74**	**7.49**	**7.39**	**7.56**	**7.50**
70	**6.24**	6.52	6.44	6.57	6.50	**6.34**	**6.84**	**6.70**	**6.90**	**6.81**
75	N/A	**5.52**	5.49	5.50	5.57	N/A	**5.58**	**5.77**	**5.83**	**5.76**
80	N/A	N/A	**4.46**	4.43	4.76	N/A	N/A	**4.56**	**4.65**	**4.64**
	Panel C: Durations When Interest Rates Decline, Male					Panel D: Durations When Interest Rates Decline, Female				
	Guarantee Period, Years					Guarantee Period, Years				
Age	20	15	10	5	0	20	15	10	5	0
55	**4.73**	**4.71**	**4.77**	**4.82**	**4.94**	**4.62**	**4.59**	**4.62**	**4.61**	**4.79**
60	**4.44**	**4.34**	**4.36**	**4.42**	**4.51**	**4.29**	**4.26**	**4.30**	**4.31**	**4.48**
65	**4.15**	**4.02**	**4.06**	**4.18**	**4.31**	**4.14**	**4.11**	**4.10**	**4.13**	**4.25**
70	**3.42**	**3.65**	**3.71**	**3.85**	**4.11**	**3.36**	**3.73**	**3.80**	**3.84**	**4.08**
75	N/A	**2.76**	**3.39**	**3.57**	**3.86**	N/A	**2.77**	**3.45**	**3.53**	**3.85**
80	N/A	N/A	**2.21**	3.35	3.59	N/A	N/A	**2.38**	**3.47**	**3.80**
	Panel E: R^2s, Male					Panel F: R^2s, Female				
	Guarantee Period, Years					Guarantee Period, Years				
Age	20	15	10	5	0	20	15	10	5	0
55	0.527	0.523	0.532	0.539	0.536	0.531	0.532	0.533	0.539	0.544
60	0.536	0.532	0.535	0.538	0.534	0.537	0.537	0.540	0.547	0.549
65	0.530	0.535	0.536	0.540	0.537	0.530	0.538	0.540	0.549	0.554
70	0.459	0.530	0.541	0.551	0.552	0.454	0.536	0.543	0.559	0.565
75	N/A	0.439	0.521	0.518	0.524	N/A	0.437	0.532	0.540	0.548
80	N/A	N/A	0.400	0.463	0.445	N/A	N/A	0.410	0.485	0.469

Note: This table displays the estimated durations of life annuities when interest rates increase or decrease, together with the model R^2s, derived from the parameter estimates of models of the form:

$$-\Delta a_i(x,g|16)/a_{i-16}(x,g) = \beta^{x,g} + \beta_{\Delta r_{i,Neg.Sign}(30|16)} \cdot \Delta r_i(30|16) \cdot I_{\Delta r,neg} + \beta^{x,g}_{\Delta r_i(30|16)} * \Delta r_i(30|16) + \epsilon_i^{x,g}.$$

The independent variables of the regressions are the lagged change in the 30-year mortgage rates measured over a period of 16 weeks, $\Delta r_i(30|16)$ and this variable interacted with a dummy variable that takes on a value of one if the interest rate decreases, $\Delta r_i(30|16) \cdot I_{\Delta r,neg}$. For remaining details, see notes to Table 6.

Robustness Checks

We perform several robustness checks to demonstrate that the results in the previous sections are not due to specific time periods or methods.

The Financial Crisis of 2008

We examine whether our results are affected by the financial crisis of 2008. Using monthly annuity quotes, Koijen and Yogo (2012) report that insurance companies sold life insurance policies and life annuities at "fire sale" prices during a few months surrounding January 2009 in order to raise capital. That is, annuity prices during that period were far lower than the contracts' actuarially fair values. Although the objective of our article is to investigate the changes in prices rather than the price levels, a period of artificially high or low prices could have an impact on our regression estimates.

Indeed, our regressions show that the regression residuals are consistently positive for December 2008. During this month, both the swap rates and the mortgage rates experienced a large decline, and so one would expect annuity prices to increase substantially.[28] The prices actually went up, but by less than predicted by the regressions, resulting in the positive residuals that we observe.[29] Note that the regression predictions are based on the estimated durations that, as shown earlier, are far below the benchmark values. This means that the increases in annuity prices in December 2008 were distinctly lower than what the benchmark durations suggests. That is, the annuity prices are especially insensitive to the movements in interest rates during December 2008. As a result, the prices in this month were farther away from their fair values, which is consistent with what Koijen and Yogo (2012) report.

However, as this unusual behavior is confined to a short period of time it should have little effect on the duration estimates. We confirm this in two ways. First, we repeat our regressions with the month of December 2008 removed from the sample. We find that while the R^2's improve, the impact on the values of the estimated durations are very small (e.g., on the order of 0.03 years for an estimated duration value of 5 years). Second, we split the sample into two subperiods—September 2004 to December 2008, and January 2009 to May 2012. The estimated durations are slightly higher for the second subperiod than for the first. However, the values in both periods are still significantly below the benchmark durations. Accordingly, we conclude that the results reported in the previous sections are not driven by the unusual annuity price behavior during the financial crisis of 2008.

[28] The 10-year swap rates declined from 3.01 percent p.a. to 2.49 percent p.a., while the 30-year mortgage rates dropped from 5.94 percent p.a. (last week of November 2008) to 5.14 percent p.a. (last week of December 2008).

[29] Note that the dependent variable in our regressions is the *negative* of the (actual) percentage changes in annuity prices. Therefore, positive residuals mean that the negative of the (actual) percentage changes are higher than predicted, which is the same as saying that the actual changes are lower than predicted.

Using the Whole Term Structure

Our use of the duration approach limits our attention to only one point in the term structure of interest rates. To check whether this limitation affects our conclusions, we run another set of regressions using all available points on the term structure of swap rates (i.e., 1, 2, 3, 5, 7, 10, and 30 years). We see no improvement in the R^2's of the regressions over the cases where only the 30-year mortgage rates are used. Therefore, it appears unlikely that our use of a single point in the term structure has a significant influence on our findings.

Results on Individual Companies: The Basic-Regression Case

Our results so far are based on the averages of annuity quotes from various annuity providers. On the one hand, the use of average numbers reduces noise from individual companies' quotes and thus makes the duration estimates more reliable. On the other hand, its use could obscure the heterogeneity of individual quotes, especially if annuity providers differ in their characteristics such as size and market shares. In this subsection, we examine whether there is cross-sectional variation in the sensitivity of annuity prices and, if so, how representative the sensitivity results based on the average quotes are.

We estimate durations using quotes from individual annuity companies that provide quotes in at least 170 weeks during the 403 weeks of the sample period (September 2004 to May 2012). The minimum number of weeks is imposed in order to ensure that the companies were sufficiently active in providing annuity quotes so that we can obtain reliable regression estimates. In total, we have 12 firms that meet this requirement. For this estimation, we use the regression in Equation (5) with the 30-year mortgage rates as the proxy for interest rates. To save space, we only report the estimates for male annuitants age 65 with no guarantee period.

The duration estimates are reported in Panel A of Table 8. It can be seen that different annuity providers have somewhat different responses to changes in interest rates. It appears that their responses can be classified into three types. First, many providers are extremely insensitive to interest rate movements over the measurement period of 1 week (i.e., durations close to 0), but their sensitivity increases when measured over longer periods, with durations of typically between 2 and 4 years (e.g., Companies A, C, F, and I). Second, a few providers are mildly sensitive to changes in interest rates over 1 week (i.e., durations of around 2 years), and become more sensitive over longer periods, with durations rising to as high as 7 years (e.g., Companies D and K). Finally, there is one provider (Company G) that adjusts its quotes much more quickly than the rest, with a duration of 5.65 years when measured over 1 week. However, its sensitivity does not increase over longer periods.

Overall, annuity providers behave similarly, and the results in the previous section based on average quotes reflect that pattern. Thus we do not believe that the low estimated sensitivity reported in the previous sections is due to the use of average quotes.

TABLE 8

Male Annuitants Age 65 With No Guarantee Period Using Annuity Quotes From Individual Insurance Companies and 30-Year Mortgage Rates

Panel A: Estimated Durations, No Asymmetry

Company	Data Period	Number of Weeks	Duration		
			$k = 1$	$k = 8$	$k = 16$
Company A	09/2007–05/2012	204	**–0.11**	**2.39**	**3.17**
Company B	09/2004–02/2008	177	**0.06**	**3.27**	**4.76**
Company C	09/2004–10/2010	255	**0.22**	**2.44**	**3.34**
Company D	09/2007–05/2012	202	**2.31**	**4.12**	**4.39**
Company E	07/2005–05/2012	312	**0.92**	**4.48**	**5.22**
Company F	09/2004–05/2012	351	**–0.05**	**2.34**	**4.16**
Company G	05/2006–05/2012	274	**5.65**	**4.43**	**5.74**
Company H	03/2005–05/2012	331	**–0.14**	**4.37**	**4.27**
Company I	09/2004–05/2012	351	**–0.11**	**2.79**	**3.6**
Company J	09/2004–05/2012	351	**–0.41**	**5.15**	**7.95**
Company K	09/2004–05/2012	354	**2.07**	**7.18**	**7.36**
Company L	02/2006–12/2011	262	**1.59**	7.17	7.43

Panel B: Estimated Durations, Asymmetric Case

	When Interest Rates					
	Increase			Decline		
Company	$k = 1$	$k = 8$	$k = 16$	$k = 1$	$k = 8$	$k = 16$
Company A	**.18**	**3.89**	**5.48**	**–0.43**	**1.40**	**1.79**
Company B	**.75**	**5.55**	**6.50**	**–0.77**	**–0.77**	**2.88**
Company C	**–.08**	**5.29**	**6.27**	**0.56**	**0.07**	**1.31**
Company D	**2.84**	**3.91**	**5.15**	**1.71**	**4.25**	**3.91**
Company E	**1.22**	**5.91**	**7.19**	**0.56**	**3.32**	**3.79**
Company F	**.24**	**4.10**	**5.33**	**–0.38**	**0.88**	**3.31**
Company G	7.09	**5.27**	8.70	4.00	**3.82**	**3.83**
Company H	**1.17**	**4.17**	**5.43**	**–1.61**	**4.52**	**3.41**
Company I	**.51**	**4.29**	**5.31**	**–0.80**	**1.55**	**2.35**
Company J	**.07**	**7.07**	7.64	**–0.95**	**3.56**	8.17
Company K	**2.45**	7.16	8.31	1.64	7.19	6.67
Company L	**1.74**	6.59	8.07	1.43	7.61	7.00

Note: This table displays the estimated durations of life annuities for a 65-year-old male with no guarantee period. The negative of the percentage changes in the annuity factors is regressed on the lagged weekly changes in the 30-year mortgage rates. Bolded coefficients denote significance at the 0.05 level, based on two-sided tests, of a difference between the estimated Duration coefficient and the theoretical value.

Results on Individual Companies: The Asymmetric Case

Is the asymmetry of the response to rate changes also robust to individual firm heterogeneity? In Panel B of Table 8, we report duration estimates for individual firms using our asymmetric response model (i.e., Equation (7)). We can see that most companies also display asymmetric responses to rate changes (i.e., the estimated durations are higher when interest rates increase than when interest rates decline). That is, when interest rates increase, their annuity prices are reduced by a greater magnitude than the increases in prices when interest rates decline. It seems unlikely, therefore, that firm heterogeneity in characteristics (e.g., asset–liability mismatches) can explain this asymmetric effect.

Results on Sortings of the Cross-Section of Companies: The Basic-Regression Case

Although results based on individual firms are consistent with results based on average quotes, there is variation in the individual responses, and we next want to find whether this heterogeneity in response is systematically related to heterogeneity in firm characteristics. To this end, we classify our sample into two groups according to annuity providers' various characteristics. These characteristics are (1) their size (as proxied by total assets, market capitalization, or sales of insurance products); (2) value versus growth, measured by the ratios of the market prices of their stocks to their book values (i.e., P/B ratios); and (3) their credit ratings. The classification into the two groups is done once a quarter based on the ranking of the values of these characteristics from the previous quarter. Except for the ranking based on credit ratings, the first group consists of firms whose values are above the median value, while the second group consists of the other half.[30] For the ranking based on credit ratings, the first group are those with superior ratings according to A.M. Best (i.e., A+ and above), which is the rating agency most used within the insurance industry, while the other group are those with ratings of excellent and below (i.e., A and below).

We then estimate durations separately for each group using Equation (5) with the 30-year mortgage rates as the proxy for interest rates. For each group, weekly annuity prices are calculated based on the averages across group members after discarding the highest and lowest quotes. The estimated durations are reported in Panel A of Table 9. As results are similar across age/guarantee periods and gender cases, we report only results for a 65-year-old male annuitant with no guarantee period.

We find that larger insurance companies (measured by total assets and market capitalization) respond slightly faster to interest rate changes than smaller providers do. The durations for larger providers are larger than the durations for smaller providers at every measurement period up to 20 weeks (including measurement periods that are not shown). The differences in durations between the two groups are not large (less than 1 year) and become smaller as the measurement period lengthens. In any case, the estimated durations for both groups are far less than the benchmark values, especially when changes are measured over a period of 1 week. When quarterly sales (of all insurance products) are used as a proxy for firm size, we get mixed results. Firms with higher sales adjust their prices more strongly than firms with lower sales do when the measurement periods are up to 11 weeks. Beyond that,

[30]Because the ranking is done every quarter, the members of each group may not remain the same from quarter to quarter. In addition, some providers did not always provide quotes, and so they could drop out of the ranking for certain quarters.

TABLE 9

Estimated Durations According to Firm Characteristics Male Annuitants Age 65 With No Guarantee Period Measurement Period $k = 1, 5, 10, 15, 20$ Weeks Using Annuity Quotes From Individual Insurance Companies and 30-Year Mortgage Rates

	Total Assets		Market Cap		Total Sales		Price-to-Book		Credit Ratings	
k	**High**	**Low**	**High**	**Low**	**High**	**Low**	**High**	**Low**	**High**	**Low**
					Panel A: The Symmetric Case					
1	**1.01**	**0.53**	**1.22**	**0.73**	**0.53**	**0.80**	**–0.15**	**0.90**	**2.30**	**1.14**
5	**4.85**	**3.96**	**5.19**	**4.03**	**5.09**	**4.05**	**5.06**	**3.99**	**3.84**	**4.07**
10	**5.34**	**4.82**	**5.60**	**4.81**	**5.41**	**5.04**	**5.74**	**4.79**	**3.09**	**4.92**
15	**5.62**	**5.62**	**5.75**	**5.45**	**5.22**	**5.97**	**6.25**	**5.50**	**3.81**	**5.50**
20	**5.85**	**5.74**	**5.92**	**5.57**	**5.29**	**6.12**	6.85	**5.67**	**4.33**	**5.46**
					Panel B: Asymmetric Case, When Interest Rates Increase					
1	**2.26**	**1.16**	**2.62**	**1.43**	**2.07**	**1.26**	**1.52**	**1.34**	**2.96**	**1.75**
5	**6.29**	**4.01**	7.59	**4.07**	7.07	**3.97**	6.91	**4.16**	**3.00**	**4.88**
10	6.88	**5.35**	7.67	**5.19**	7.28	**5.36**	8.90	**5.20**	**4.21**	**6.14**
15	8.29	**6.55**	8.25	**6.43**	7.99	**6.55**	7.76	**6.16**	**5.02**	**6.93**
20	7.35	**6.60**	7.31	**6.65**	6.53	6.91	5.86	**6.58**	**5.13**	7.20
					Panel C: Asymmetric Case, When Interest Rates Decrease					
1	**–0.36**	**–0.16**	**–0.33**	**–0.05**	**–1.18**	**0.30**	**–2.01**	**0.40**	**1.58**	**0.46**
5	**3.62**	**3.91**	**3.13**	**3.99**	**3.39**	**4.12**	**3.47**	**3.84**	**4.47**	**3.37**
10	**4.08**	**4.39**	**3.91**	**4.49**	**3.88**	**4.78**	**3.15**	**4.46**	**2.24**	**3.92**
15	**3.57**	**4.91**	**3.84**	**4.70**	**3.09**	5.53	5.10	**4.99**	**2.98**	**4.40**
20	**4.95**	**5.21**	**5.08**	**4.92**	**4.55**	**5.65**	7.44	**5.12**	**3.95**	**4.41**

Note: This table displays the estimated durations of life annuities for a 65-year-old male with no guarantee period. The estimations are done using firm characteristics where individual insurance companies are divided into two groups according to firm characteristics. The negative of the percentage changes in the annuity factors in the average annuity factors of each group of firms is regressed on the lagged weekly changes in the 30-year mortgage rates. Market Cap is firms' market capitalization. Total Sales is the combined sales of firm's insurance products. Price-to-Book is the ratio between shares' market prices and book values. Credit Ratings is ratings by A.M. Best. Bolded coefficients denote significance at the 0.05 level, based on two-sided tests, of a difference between the estimated duration coefficient and the theoretical value.

the reverse is true. The differences between the two groups, however, are less than 1 year for all measurement periods. Interestingly, "growth" firms (those with higher market-price-to-book-value ratios) have higher estimated durations than "value" firms (those with lower ratios) do. Finally, companies with higher credit ratings adjust their prices faster than lower-rated companies do. This is evidenced by the fact that the estimated durations for higher-rated companies are greater than those of lower-rated companies over short measurement periods ($k \leq 4$ weeks). On the other hand, when measured over longer periods ($k \geq 7$ weeks), the estimated durations of annuities offered by lower-rated companies are larger.

We also rank insurance companies every week based on their annuity quotes in the previous week. Firms whose quotes are greater than the median quote go into the first group, while the rest go into the second group. We want to see whether firms that are more aggressive in their pricing also exhibit more sensitivity to interest rate movements. Since this ranking is done based on the previous week's quotes, we can only estimate price sensitivity over the period of 1 week in order to avoid endogeneity issues. The results (not shown) are virtually the same between the two groups. That is, we find no evidence that more aggressive firms are also more responsive to changes in interest rates.

Overall, the above cross-sectional examination of annuity prices uncovers some differences in how individual annuity providers behave. Firms with larger size and/or higher credit ratings are slightly quicker to adjust to changes in market interest rates. This is consistent with the argument that large firms are in a better competitive position and so are more flexible in adjusting their prices. It is also consistent with the findings in Hannan and Berger (1991) that larger banks are less rigid in changing their deposit rates. The results also show that although lower-rated providers are slower to respond to interest rate movements, their adjustments are virtually indistinguishable when measured over longer periods. A similar pattern of differences in response is observed when firms are classified according to their sales. It should be noted that regardless of the firm characteristics that we use in the rankings, the differences between the two groups of firms are generally small.

Results on Sortings of the Cross-Section of Companies: The Asymmetric Case

Lastly, we examine the asymmetric responses when firms are sorted into two groups and check whether the heterogeneity in firm characteristics has any effect on the responses when rates increase versus decline. The results are reported in Panels B and C of Table 9. It can be seen that the asymmetric responses are present regardless of what characteristic we use to separate the firms. More importantly, it appears that the two groups of firms behave differently depending on the direction of the interest rate movements. When rates increase, larger firms (by assets, market caps, or sales) adjust (i.e., reduce) their prices faster and more completely than smaller firms do. However, the opposite is true when rates decline. In other words, larger firms seem to move prices in a way that is more attractive to customers. This reinforces the idea that larger firms have more flexibility in price adjustments, and they use this flexibility to their advantage.

A similar behavior is observed between growth and value firms. When rates increase (decline), growth firms change their prices more (less) quickly and completely than value firms do. Finally, we find that higher-rated firms react more quickly than lower-rated firms do (i.e., having higher durations over very short measurement periods) regardless of the direction of the interest rate movements.

Conclusion and Implications

In this article, we tested whether annuity prices promptly and fully adjust to changes in interest rates. The prompt and full response of market prices to interest rate changes is a standard assumption that is made, implicitly or explicitly, in most of the annuity-related scholarly literature.

Our main result is that annuity price changes do not behave as assumed. Rather, prices gradually adjust over a period of several weeks to respond to interest rate changes that occur over the same number of weeks. We conjecture that this result is due to insurance company's desire to rebalance their assets/liabilities portfolios, since they can from time to time be unbalanced (i.e., mismatches of durations of assets and liabilities). They may deliberately offer relatively uncompetitive annuity prices or delay price adjustments in order to allow for time to rebalance.

Unlike the traded bond market, in which such a result would be inconceivable, market for life annuities does not allow arbitrage. Once the annuity is purchased it is very difficult to sell it back to the insurance company, and a short position in a life annuity is impossible. So, in some sense it is not surprising to find that the prices behave quite differently from traded fixed-income products.

In addition, we find that the response of annuity prices to changes in interest rates is asymmetric. Annuity prices react more rapidly and with greater sensitivity to an increase in interest rates than to a decrease in interest rates. That is, when interest rates increase, insurance companies reduce their annuity prices (by increasing the amount of monthly annuity income per the same premium amount) more quickly and in a larger magnitude than what they do in the opposite direction when rates decline. In other words, annuity rates (which are inversely related to annuity prices) are more rigid downward than upward. This is unexpected, but our findings are consistent with the customer reaction hypothesis whereby insurance companies may fear a negative reaction from potential annuity buyers if annuity rates are reduced quickly, in response to interest rate changes.

These results are robust with respect to annuitant ages, lengths of guarantee periods, choice of reference interest rates, subsets of annuity providers, and the subperiods of the sample. As part of our robustness tests, we uncover a small degree of heterogeneity in the responses of different individual insurance companies. Firms with larger size and/or higher credit ratings are slightly quicker to adjust to changes in market interest rates. Lower-rated providers, however, adjust in a slightly more complete manner when measured over longer periods. A similar pattern of differences in response is observed when firms are classified according to their sales. Nevertheless, these cross-sectional differences are not large enough to affect our overall findings.

Our findings have several implications on a growing financial economics literature related to life and pension annuities. The fact that prices are slow to adjust to new interest rate information suggests that the timing of the purchase can affect the individual's optimal allocation decisions and the magnitude of welfare gains from annuitization. This is particularly important considering that an annuity purchasing decision is irreversible and typically involves a substantial portion of an individual's wealth. In addition, our results imply that the money's worth of an annuity can vary depending on the timing of the calculations (even if every other factor remains the same). Similarly, caution should be exercised when attempting to infer mortality expectations from observed annuity prices at a single point in time. We leave the quantification of the impact of these annuity pricing puzzles on the money's work calculation, mortality expectations, and so on, for further work.

APPENDIX A: GENERAL DERIVATION OF ANNUITY FACTOR AND DURATION

In the body of the article, we derived an expression for the duration of a life annuity assuming the (pricing) yield curve was flat and equal to a constant r. In that simple case, the duration was defined as the (negative) derivative of the annuity factor $a(x, r)$ with respect to the pricing rate r, and then scaled by the same annuity factor $a(x, r)$. In this appendix, we derive a more general duration measure D, assuming the annuity is priced with a *nonconstant* yield curve denoted by r_t. The duration is properly defined in terms of a parallel shift in the yield curve. Arguably, this is more realistic compared to assuming a constant rate.

Our objective is not to (better) price life annuities. Rather, it is to show that *even* under a more general definition of interest rate sensitivity, the theoretical values for the duration of a life annuity are very similar to the values obtained under the more restrictive (constant) case. In other words, regardless of how exactly the interest rate sensitivity is defined for a life annuity, the theoretical values are much larger than the empirically observed ones.

In general the annuity factor can be represented as follows:

$$a(x, g, \vec{\mathbf{r}}) = \int_0^\infty \mathrm{e}^{-(r_t)t} p(x, t)\, \mathrm{d}t, \tag{A1}$$

where r_t is the entire yield curve, and $\vec{\mathbf{r}}$ is short-hand notation for the function itself. The survival probability function $p(x, t)$ is defined in the following way:

$$p(x, t) = \mathrm{e}^{-\int_0^t \lambda_s^g \, \mathrm{d}s}, \tag{A2}$$

where λ_s^g is a (truncated) mortality rate defined by:

$$\lambda_s^g = \begin{cases} 0 \text{ for } s \le g \\ \lambda_s \text{ for } s > g. \end{cases} \tag{A3}$$

Using this notation, g is the guarantee (certain) period in years and λ_s is any continuous mortality rate. Since payments are guaranteed for g years, the effective mortality rate is zero until time $s = g$.

The derivative of the annuity factor with respect to a parallel shift ε in the yield curve can be defined and expressed as follows:

$$\frac{\partial a(x, g, \vec{\mathbf{r}}+\varepsilon)}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon} \int_0^\infty \mathrm{e}^{-(r_t+\varepsilon)t} p(x,t)\,\mathrm{d}t = \int_0^\infty -t\mathrm{e}^{-(r_t+\varepsilon)t} p(x,t)\,\mathrm{d}t. \tag{A4}$$

The ability to switch the integral and the derivative sign follows directly from Fubini's Theorem, since our functions $p(x, t)$ and $(r_t + \varepsilon)t$ are well behaved and satisfy the necessary differentiability conditions.

Now, if the interest rate (yield) curve r_t is continuous over the interval $(0, t)$, then the composite function $r_t t$ is differentiable and by the fundamental theorem of calculus can be written as:

$$r_t t = \int_0^t \xi_s\,\mathrm{d}s. \tag{A5}$$

As an example, consider the interest rate function $r_t = r - c(t+1)^{-1}$. It starts at a value of $(r - c)$ for maturity zero, and then asymptotes to r as $t \to \infty$. The function is obviously continuous, and the product with time is $r_t t = rt - ct(t+1)^{-1}$. So, the function $\xi_s := r - c((t+1)^{-1} - t(t+1)^{-2})$.

A by-product of this decomposition is that the derivative of the annuity factor with respect to ε can now be expressed as:

$$\frac{\partial a(x, g, \vec{\mathbf{r}}+\varepsilon)}{\partial \varepsilon} = \int_0^\infty -t\mathrm{e}^{-\varepsilon t}\mathrm{e}^{-\int_0^t \xi_s\,\mathrm{d}s} p(x,t)\,\mathrm{d}t = \int_0^\infty -t\mathrm{e}^{-\varepsilon t}\mathrm{e}^{-\int_0^t (\xi_s+\lambda_s^g)\,\mathrm{d}s}\,\mathrm{d}t \tag{A6}$$

$$= \int_0^\infty -t\mathrm{e}^{-\varepsilon t}\hat{p}(x,t)\,\mathrm{d}t, \tag{A7}$$

where the new survival probability curve $\hat{p}(x, t)$ is defined structurally as before, but with a new underlying hazard rate:

$$\hat{\lambda}_s^g = \begin{cases} \xi_s & \text{for } s \le g \\ \lambda_s + \xi_s & \text{for } s > g. \end{cases} \tag{A8}$$

Intuitively, this is a mortality model in which the mortality rate is increased by the relevant interest rate, which makes them older. All else being equal, the (simple) duration will be lower, but will satisfy the same structural equation derived within the body of the article, but with different mortality rate parameters.

At this point we (finally) let $\varepsilon \to 0$, so that $e^{-\varepsilon t} \to 1$ and the duration formally becomes:

$$D(x, g) := \left.\frac{-\partial a(x, g, \vec{\mathbf{r}}+\varepsilon)}{\partial \varepsilon} \frac{1}{a(x, g, \vec{\mathbf{r}})}\right|_{\varepsilon=0} = \frac{1}{a(x, g, \vec{\mathbf{r}})} \int_0^\infty t\hat{p}(x,t)\,\mathrm{d}t. \tag{A9}$$

This expression can be computed numerically (fairly easily.) For example, consider the following yield curve model $r_t = 0.04314 - (0.0275)(t+1)^{-1}$, which is a nonconstant upward sloping yield curve. The zero-maturity rate is $r_0 = 0.04314 - 0.0275 = 0.01564$, which is 1.564 percent. The 10-year maturity rate is $r_{10} = 0.04314 - (0.0275)(11)^{-1} = 0.04064$, which is 4.06 percent—the same rate we used within the body of the article. The asymptotic rate is $r_\infty = 4.314$ percent This is *ad hoc*, but our point is as follows.

Assuming a $x = 65$ and $x = 75$-year-old female with Gompertz mortality (parameters $m = 92.63$ and $b = 8.78$), and the above interest rate curve, the annuity factor's are:

$$a(65, 0, \vec{\mathbf{r}}) = \$14.5577$$

$$a(75, 0, \vec{\mathbf{r}}) = \$10.9909$$

and the generalized duration will be:

$$D(65, 0) = 10.5599 \tag{A10}$$

$$D(75, 0) = 7.9016. \tag{A11}$$

which are very close to, but slightly lower than, the values presented in Table 1, which assumed a flat 4.064 percent pricing curve. In other words, a more robust yield curve model does not result in (much) lower durations and cannot explain our empirical findings.

APPENDIX B: SOURCE OF ANNUITY DATA

Our raw data were obtained using a three-step process. In the first step we utilized the services of CANNEX Financial Exchanges, which is a data vendor with an ongoing business relationship with (most) U.S. insurance companies that sell and market life annuities. CANNEX has the ability to obtain any prespecified annuity quote for a wide range of hypotheticals, such as, age, gender, period certain, and so on. Financial advisors, brokers, and planners use CANNEXs services to obtain current quotes.

From the insurance company's perspective, the numbers quoted are the prices they would be willing to offer for any given annuity. This is the equivalent of calling a foreign exchange dealer and asking them their bid/ask price for a given quantity of a particular currency. The CANNEX system effectively contacts all insurance companies at the exact same time inquiring about the price of many different annuities. Insurance companies would be obligated to honor those quotes, even though they do not know that the inquiry was a hypothetical query. In the second stage, CANNEX sends this (large) data file to the QWeMA Group (a software company) on a weekly basis, which the QWeMA Group checks for internal consistency and then stores these numbers. The QWeMA Group constructs indices for annuity prices based on the averages of these (thousands of quotes). The QWeMA Group stores the numbers in an easily accessibly manner and has made summaries and various time series available to researchers. Finally, we obtained the raw company-by-company data from the QWeMA Group and extracted the relevant age by guarantee matrix we required for our analysis. We therefore refer to this data set as the CANNEX-QWeMA data set and gratefully acknowledge both companies assistance in this research.[31]

[31]Disclosure Note: As of early 2014 the QWeMA Group was acquired by CANNEX Financial and they are essentially one company. In addition one of the authors has a financial interest in CANNEX.

References

Allen, M. T., R. C. Rutherford, and M. K. Wiley, 1999, The Relationships Between Mortgage Rates and Capital-Market Rates Under Alternative Market Conditions, *Journal of Real Estate Finance and Economics*, 19(3): 211-221.

Ameriks, J., A. Caplin, S. Laufer, and S. Van Nieuwerburgh, 2011, The Joy of Giving or Assisted Living? Using Strategic Surveys to Separate Public Care Aversion From Bequest Motives, *Journal of Finance*, 66(2): 519-561.

Benartzi, S., A. Previtero, and R. H. Thaler, 2011, Annuitization Puzzles, *Journal of Economic Perspectives*, 25(4): 143-164.

Bodie, Z., 1990, Managing Pension and Retirement Assets: An International Perspective, *Journal of Financial Services Research*, 4(4): 419-460.

Boyce, S., and R. A. Ippolito 2002, The Cost of Pension Insurance, *Journal of Risk and Insurance*, 69(2): 121-170.

Brown, J. R., O. S. Mitchell, and J. M. Poterba, 2001, The Role of Real Annuities and Indexed Bonds in an Individual Accounts Retirement Program, in: J. Y. Campbell and M. Feldstein, eds., *Risk Aspects of Investment-Based Social Security Reform* (Chicago: University of Chicago Press), pp. 321-370.

Brown, J. R., and D. W. Wilcox 2009, Discounting State and Local Pension Liabilities, *American Economic Review*, 99(2): 538-542.

Campbell, J., and L. Viciera, 2002, *Strategic Asset Allocation: Portfolo Choice for Long Term Investors* (New York: Oxford University Press).

Cannon, E., and I. Tonks, 2005, Survey of Annuity Pricing, Department for Work and Pensions Research Report No. 318.

Cannon, E., and I. Tonks, 2009, Money's Worth of Pension Annuities, Department for Work and Pensions Research Report No. 563.

Cocco, J., F. Gomes, and P. Maenhout, 2005, Portfolio Choice Over the Life-Cycle, *Review of Financial Studies*, 18: 491-533.

Davidoff, T., 2009, Housing, Wealth and Annuities, *Journal of Risk and Insurance*, 76(1): 31-52.

Davidoff, T., J. R. Brown, and P. A. Diamond, 2005, Annuity and Individual Welfare, *American Economic Review*, 95(5): 1573-1590.

Dawson, P., K. Dowd, A. J. G. Cairns, and D. Blake, 2010, Survivor Derivatives: A Consistent Pricing Framework, *Journal of Risk and Insurance*, 77(3): 579-596.

Deng, Y., P. L. Brockett, and R. D. MacMinn, 2012, Longevity/Mortality Risk Modeling and Securities Pricing, *Journal of Risk and Insurance*, 79(3): 697-721.

Dickson, D. C. M., M. R. Hardy, and H. R. Waters, 2009, *Acturial Mathematics for Life Contingent Risks* (Cambridge, UK: Cambridge University Press).

Dushi, I., and A. Webb, 2004, Annuitization: Keeping Your Options Open, Boston College Center for Retirement Research Working Paper No. 2004-04.

Feldstein, M., and E. Ranguelova 2001, Individual Risk in an Investment-Based Social Security System, *American Economic Review*, 91(4): 1116-1125.

Finkelstein, A., and J. M. Poterba, 2004, Selection Effects in the United Kingdom Individual Annuities Market, *Economic Journal*, 112: 28-50.

Finkelstein, A., J. M. Poterba, and C. Rothschild, 2009, Redistribution by Insurance Market Regulation: Analyzing a Ban on Gender-Based Retirement Annuities, *Journal of Financial Economics*, 91: 35-58.

Friedman, B. M., and M. J. Warshawsky, 1990, The Cost of Annuities: Implications for Saving Behavior and Bequests, *Quarterly Journal of Economics*, 105(1): 135-154.

Hannan, T. H., and A. N. Berger, 1991, The Rigidity of Prices: Evidence From the Banking Industry, *American Economic Review*, 81(4): 938-945.

Haybittle, J. L., 1998, The Use of the Gompertz Function to Relate Changes in Life Expectancy to the Standardized Mortality Ratio, *International Journal of Epidemiology*, 27(5): 885-889.

Horiuchi, S., and A. J. Coale, 1982, A Simple Equation for Estimating the Expectation of Life at Old Ages, *Population Studies*, 36(2): 317-326.

Horneff, W. J., R. H. Maurer, O. S. Mitchell, and M. Z. Stamos, 2009, Asset Allocation and Location Over the Life Cycle With Investment-Linked Survival-Contingent Payouts, *Journal of Banking & Finance*, 33(9): 1688-1699.

Hull, J., M. Predescu, and A. White, 2005, Bond Prices, Default Probabilities, and Risk Premiums, *Journal of Credit Risk*, 1(2): 53-60.

Inkmann, J., P. Lopes, and M. Michaelides, 2011, How Deep Is the Annuity Market Participation Puzzle? *Review of Financial Studies*, 24(1): 279-319.

Ippolito, R. A., 2002, Replicating Default Risk in a Defined-Benefit Plan, *Financial Analyst Journal*, 58(6): 31-40.

James, E., and X. Song, 2001, Annuities Markets Around the World: Money's Worth and Risk Intermediation, Center for Research on Pensions and Welfare Working Paper.

Koijen, R. S. J., and M. Yogo, 2012, The Cost of Financial Frictions for Life Insurers, NBER Working Paper No. 18321.

Lee, R. D., and L. R. Carter, 1992, Modeling and Forecasting U.S. Mortality, *Journal of the American Statistical Association*, 87: 659-671.

Maurer, R., O. S. Mitchell, R. Rogalla, and V. Kartashov, 2013, Lifecycle Portfolio Choice With Systematic Longevity Risk and Variable Investment-Linked Deferred Annuities, *Journal of Risk and Insurance*, 80(3): 649-676.

Milevsky, M. A., and V. R. Young 2007a, Annuitization and Asset Allocation, *Journal of Economic Dynamics and Control*, 31(9): 3138-3177.

Milevsky, M. A., and V. R. Young, 2007b, The Timing of Annuitization: Investment Dominance and Mortality Risk, *Insurance: Mathematics and Economics*, 40(1): 135-144.

Mitchell, O. S., J. M. Poterba, M. J. Warshawsky, and J. R. Brown, 1999, New Evidence on the Money's Worth of Individual Annuities, *American Economic Review*, 89(5): 1299-1318.

Modigliani, F., and R. Brumberg, 1954, Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data, in: K. K. Kurihara, ed., *Post Keynesian Economics* (New Brunswick, NJ: Rutgers University Press), pp. 388-436.

Neumark, D., and S. Sharpe, 1992, Market Structure and the Nature of Price Rigidity: Evidence From the Market for Consumer Deposits, *Quarterly Journal of Economics*, 107(2): 657-680.

Newey, W. K., and K. D. West 1994, Automatic Lag Selection in Covariance Matrix Estimation, *Review of Economic Studies* 61: 631-653.

Novy-Marx, R., and J. Rauh, 2011, Public Pension Promises: How Big Are They and What Are They Worth? *Journal of Finance*, 66(4): 1211-1249.

Payne, J. E., 2006, The Response of the Conventional Mortgage Rate to the Federal Funds Rate: Symmetric or Asymmetric Adjustment? *Applied Financial Economics Letters*, 2(5): 279-284.

Philips, T. J., and G. S. Becker, 1998, Old-Age Longevity and Mortality-Contingent Claims, *Journal of Political Economy*, 106(3): 551-573.

Poterba, J. M., 1997, The History of Annuities in the United States, NBER Working Paper 6004.

Promislow, S. D., 2011, *Fundamentals of Actuarial Mathematics*, 2nd edition (New York: John Wiley & Sons).

Reitano, R. R., 1992, Non-Parallel Yield Curve Shifts and Immunization, *Journal of Portfolio Management*, 18(3): 36-43.

Richard, S., 1975, Optimal Consumption, Portfolio and Life Insurance Rules for an Uncertain Lived Individual in a Continuous Time Model, *Journal of Financial Economics*, 2: 187-203.

Rotemberg, J. J., 1982, Sticky Prices in the United States, *Journal of Political Economy*, 90: 1187-1211.

Roth, H. L., 1988, Volatile Mortgage Rates—A New Fact of Life? *Federal Reserve Bank of Kansas City Economic Review*, 16-28.

Scott, S., J. G. Watson, and W. Hu, 2011, What Makes a Better Annuity? *Journal of Risk and Insurance*, 78(1), 213-244.

Sun, W., and A. Webb, 2011, Valuing the Longevity Insurance Acquired by Delaying Social Security, *Journal of Risk and Insurance*, 78(4): 907-929.

Sundaresan, S., and F. Zapatero, 1997, Valuation, Optimal Asset Allocation, and Retirement Incentives of Pension Plans, *Review of Financial Studies*, 10(3): 631-660.

Turra, C. M., and O. S. Mitchell, 2004, The Impact of Health Status and Out-of-Pocket Medical Expenditures on Annuity Valuation, NBER Working Paper.

Yaari, M. E., 1965, Uncertain Lifetime, Life Insurance and the Theory of the Consumer, *Review of Economic Studies*, 32: 137-150.